ORCHESTRATING BOTH STUDENT AUTHORITY AND ACCOUNTABILITY TO THE DISCIPLINE WHEN GUIDING STUDENTS PRESENTING A PROOF

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Teachers often find it hard to balance between justice to the students' input and leading the class towards the decided goal. We focus on how the teacher orchestrates the balance between whole class student authority and accountability to the discipline. In the case a student presents the result from group work but at some point needs help. The teacher de-personalises the discussion and directs the class' attention to the subject and not to individual students. Thereby, the class is treated as a community with a shared authority. By the end, collective learning has taken place.

Keywords: Classroom discourse, Communication, High school education, Reasoning and proof

Introduction

The teacher's responsibility is to lead classroom discussions that build on student thinking and guides the class to "strike an appropriate balance between giving students authority over their mathematical work and ensuring that the work is held accountable to the discipline" (Stein et al., 2008, p. 332). Teacher-class discussions were analysed as acquisition of mathematical knowledge (Prediger et al., 2015). Less attention has been paid to how teachers manage situations where students' presentations fail to present the group's end result or provide understandable explanations. How can the teacher respond without simply taking over the explanation? This paper focuses on a case where the teacher intervenes during a student's presentation and manages to give clear responses without "outshining" the students.

Theoretical framework

Participation

The *participationist perspective* denotes all approaches where learning is conceptualized as participation in classroom discourses and collectively implemented activities (Sfard, 2008). Learning mathematics is a process of enculturation into mathematical practices including discursive practices and how they are interactively established in classroom micro-cultures. Mathematical practices capture collective mathematical development and describe interactively established ways of joint action in mathematics classrooms. The participation perspective intertwines *discursive participation*, taking part in discourse practices according to discursive norms and *epistemic participation*, taking part in the joint epistemic processes of knowledge constitution (Erath et al., 2018).

Discursive approach and collective learning

Cobb et al. (2011) write about the collective learning of the classroom community as the evolution of classroom mathematical practices. In line with this, Lerman (2002) outlines the principles of a cultural, discursive psychology, where learning is an initiation into the practices of school mathematics including learning to speak mathematically. The teacher has a vital role in showing what is approved within the discourse, i.e. the accountability to the discipline. Furthermore: "interactions should not be seen as windows on the mind but as discursive contributions that may pull others forward into their increasing participation in mathematical speaking/thinking" (Lerman, 2002, p. 89), which is in line with Sfard's (2008) view of learning as a combination of acquisition and participation.

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Gravemeijer (2004, p. 126) points to “the proactive role of the teacher in establishing an appropriate classroom culture, in choosing and introducing instructional tasks, organising group work, framing topics for discussion, and orchestrating discussion”. In line with this, Stein et al. (2008, p. 320) emphasise the importance of “whole-class discussions in which the teacher actively shapes the ideas that students produce to lead them to more powerful, efficient, and accurate mathematical thinking.” In the Discursive Approach by Sierpinska (2005), the teachers’ role in classroom conversations is similarly characterised by an obligation to lead the discussion in the direction of relevant mathematical ideas and themes. In line with this, we introduced the term *captivating dialogue* (Andresen & Dahl, 2018) to situations where students are progressively initiated into the practices of school mathematics through a whole class discussion.

Research question

How can a teacher’s orchestrate a balance between student authority and accountability to the discipline while guiding the presentation of students’ group work?

Methodology

The data consisted of video recordings (30 hours) of teaching during the autumn of 2013 in eight Norwegian upper secondary classrooms as part of the EU research project *KeyCoMath* about students’ strategies for creative problem-solving (Andresen, 2015, 2018). The aim of the project was to develop and study teaching that encourages students’ inquiry, and intellectual autonomy. The teachers were experienced teacher who volunteered to develop exploratory mathematics tasks to their own classes with the purpose of stimulating student inquiry. This paper focuses on one sequence (6 minutes, 28 seconds, translated to English) and discusses the interactions between the student at the blackboard, the rest of the students, and the teacher. The utterances are not analysed as isolated events but as they occur in a context of sequential utterances and the analysis does not evaluate each utterances in terms of whether or not they are evidence of learning, as we perceive learning as a result of a combination of a series of events.

Data and analysis

Tina teaches a mathematics class of 24 students from a larger town. The excerpt is from the final lesson of a ‘Mathematics Day’ where the students worked in groups with tasks from ‘Proofs without words’ (Nelsen, 1993). Each task asks for an explanation of the connection between its figure and its formula. This type of tasks was novel to the students. The excerpt shows a student (Ingvild) who volunteered to demonstrate her group’s solution to the task in Figure 1.

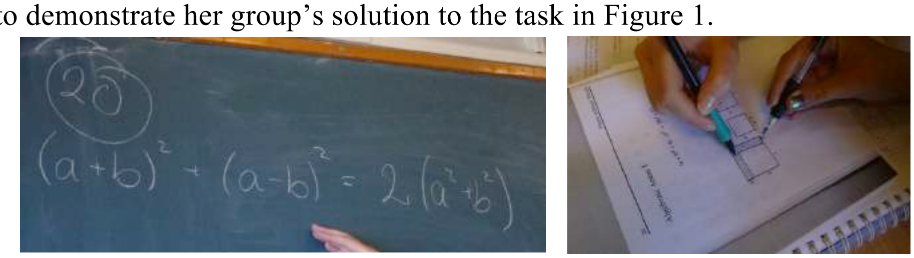

Figure 1: Task as shown on the blackboard (left), student work from book (right)

Ingvild appears calm and relaxed. The task and a drawing of the square is seen on both the blackboard and the book (see Figure 1, left & right & Figure 2, left side of the left equation).

Ingvild: We are supposed to deduct something, right?
Ingvild: But okay. I was thinking ... it was ... [looks at Tina]
Tina: They [the class] are the ones you are supposed to explain it to

Ingvild: Okay [smiling]. It is ($a + b$)2 because ehh we have a times a here [points to the smaller square inside the bigger square and writes a^2 on this square] and ehh no [looks at Tina and appears doubtful]
Tina: Help [aimed at the class]
Tina: There is a lot of help [several hands have been raised]

After 41 seconds Ingvild hesitates and looks appealing at Tina who stands next to the blackboard. Two points of interest: i) Tina gives the *authority to Ingvild* to explain something to the whole class and not only to Tina. Here, Tina emphasizes the *class' understanding* rather than checking the correctness of the result or *Ingvild's understanding*. ii) Ingvild gets stuck almost immediately, but Tina does not take over the explanation but directs Ingvild's attention to the class and requires Ingvild to get help from the class ("There is a lot of help"). By that, Tina assigns the *authority to the class* and encourages interaction between Ingvild and the class.

Next, different students in the class contribute to the task's solution, and one student says: "On the one side it is $a + b$ and the same down. Therefore, in a way it becomes $a + b$ times $a + b$ and we can write this as $(a + b)^2$ …". To which Ingvild responds: "But where does a^2 come from [points at a^2 in the square]". Several students then start to explain at the same time. After 1 minute and 18 seconds Tina interrupts and says: "Someone needs to come up [to the blackboard] and explain it". No one volunteers but several students provide explanations from their seat. Ingvild frequently replies "Hmm", nods and points at the mentioned places on the figure. In our interpretation, the class accepts the authority given by Tina and willingly participates in explaining. Ingvild can follow the suggestions and although she failed to explain the task on her own, she does not appear timid by the situation and her peers do not appear to ridicule her.

After 2 minutes and 20 seconds, Ingvild takes over again and draws the second square with side length $a - b$ (see Figure 2, left) and explains:

Ingvild: And it becomes $a - b$ and $a - b$ [points to each side and then looks at Tina]
Tina: Hm-hm [accepting sound]

This time, Tina does not ask the class for help, but indicates that Ingvild is on track, which we interpret as Tina showing accountability to the discipline by focusing on mathematical content. Until 4 minutes and 48 seconds into the recording, Ingvild draws the rest of the figures seen in Figure 2 while she and some students are discussing what to do. Next, Ingvild hands out the chalk with a happy smile, as if she thinks she has finished. But Tina intervenes:

Tina: I do not quite understand it all. [Tina points to a^2 and b^2]. But these ones? [pointing at the two strips on the left side of the right equality while turning towards the class]

Until 6 minutes and 17 seconds into the recording, Tina exchanges with different students, including Ingvild. The intriguing part of the reasoning, illustrated in Figure 2 (right side), appears to be that the dark rectangles (a times b) 'overlap' in the second square on the left side, and, therefore, must be added (as b times b) to the last square on the right side. Tina asks questions like: "How big is this piece?" to make sure that all the areas in the squares on the left side of the equality sign are represented at the right side of the equality sign. Then Tina asks: "Is it correct?" and concludes by saying: "So based on this where we have drawn – thank you [to Ingvild] – two squares". At this point the class apparently impulsively applauses while Ingvild returns to her seat. Visible signs of agreement are students' nodding and confirming answers, and nobody asks more questions although the atmosphere is forthcoming.

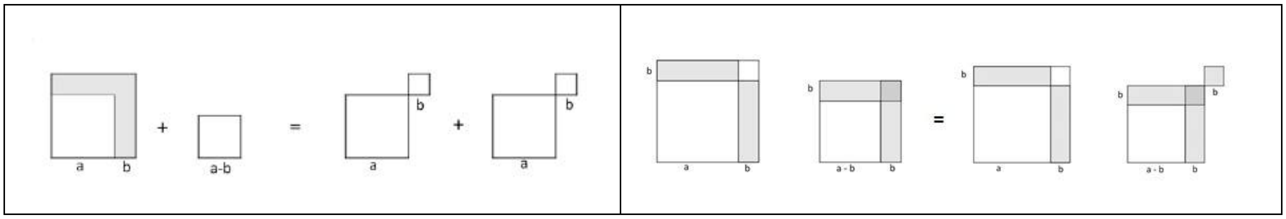

Figure 2: Figures from the task (left), figures from the blackboard (right)

Discussion

Accountability and authority

In our interpretation, Tina shows accountability to the discipline by spending almost one fourth of the sequence's time (towards the end) to make sure that all figures to the right have been covered by the figures to the left. The accountability is balanced with student authority as Tina directs focus of attention to the interaction between the whole class and the subject on the blackboard rather than focusing on the interaction between Ingvild and herself. Further, Tina gives the authority to the community of students when she requests help. We also see that teacher authority is not the same as teacher monologue, Tina orchestrates is in complete control even though she is at the background most of the time.

Classroom culture, participation and collective learning

Ingvild had volunteered and does not exhibit discomfort when she gets stuck. Tina thanks Ingvild during the conclusion of the sequence, and the class applauds even though it was not a brilliant presentation. This shows a classroom culture with ample space for student authority and for discussion. We also see that Tina does not only focus on student authority. Stein et al. (2008) describes that sometimes a focus on student thinking is perceived to imply that the teacher "must avoid providing any substantive guidance at all" (p. 316). In Tina's case, providing substantive guidance is not in itself a contradiction to a student-centred classroom culture. Tina manages the balance and establishes a classroom culture in which the students through discursive participation create the basis for collective learning. By the end of the sequence, collective learning (Cobb et al., 2011) has taken place and *the class* knows the solution. Tina ensures that the class is on the path which is in line with Sierpinska's (2005) views of the role of the teacher as someone who has the responsibility of leading a class in a relevant direction.

Conclusions

In this paper we address how a teacher orchestrates the balance between accountability to the discipline and authority to the students. We focus on students in the classroom as a group and analyse a sequence from a lesson where a student presents the result of group work. The teacher avoids the face-to-face communication with the presenting student, which could otherwise have been the teacher's choice of action when a student gets stuck in the explanation. Thereby the teacher manages to insist on the inclusion of the whole class into the discussion. Furthermore, the teacher avoids taking over from the student and giving the explanations using her authority. Rather, the teacher encourages and supports the rest of the class to develop the appropriate explanations. In our interpretation, the students' authority therefore remains acknowledged together with relevant mathematics that is held accountable to the discipline.

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