COMPUTATIONAL THINKING PRACTICES AS A FRAME FOR TEACHER ENGAGEMENT WITH MATHEMATICS CURRICULUM MATERIALS

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Teachers routinely make adaptations to their mathematics curriculum materials as they plan and enact lessons. In this paper, I explore how encouraging two elementary teachers to examine their mathematics curriculum materials through the lens of computational thinking practices – abstraction, debugging, and decomposition – supported them in adapting tasks from their curriculum materials in ways that raised the cognitive demand.

Keywords: Computational Thinking; Curriculum; Elementary School Education

Background and Purpose of the Study

Mathematics curriculum materials (CMs) can serve as supports for teachers in creating high-quality mathematics instruction (Stein & Kaufman, 2010; McGee, Wang, & Polly, 2013). One way CMs can act as a support is by providing tasks with high cognitive demand (Stein, Smith, Henningsen, & Silver, 2000) or starting points for such tasks. For the potential of this support to be realized in practice, teachers must learn ways of interacting with CMs that allow them to thoughtfully choose among tasks and adapt them in ways that support students' engagement with productive mathematics (Brown, 2009). Existing research has revealed teachers differ in the specific strategies they use to approach CMs (Remillard, 2012; Sherin & Drake, 2009), and not all such strategies result in instruction that maintains high cognitive demand for students (e.g., Amador, 2016). On the other hand, a number of studies have supported the notion that teachers can create instruction that maintains high cognitive demand when they engage with CMs through lenses related to big mathematical ideas (Stein & Kaufman, 2010), student thinking and knowledge (Choppin, 2011; Grant et al., 2009), and the connections between these two elements (Drake et al., 2015). In this paper, I present a post-hoc analysis of how two teachers adapted tasks when reviewing a lesson in their CMs through the lens of three computational thinking (CT) practices: decomposition, debugging, and abstraction. In particular, I focus on how these three CT practices supported teachers in transforming low cognitive demand tasks presented in their curriculum materials into tasks of higher cognitive demand by helping to focus teachers' attention on the big mathematical ideas of the lessons and students' potential strategies.

Conceptual Framework

This study utilized the mathematical task framework, which has two parts. First, Smith and Stein (1998) developed four cognitive demand categories for mathematics tasks. Two categories —Doing Mathematics and Procedures with Connections—are high cognitive demand because they engage students in thinking about mathematical concepts and relationships. The others—Procedures without Connections and Memorization—are low cognitive demand because they focus on use of procedures and correct answers. Second, Smith, Grover, and Henningsen (1996) argued any task passes through three phases when used in instruction: (1) the task as it appears in instructional resources, (2) the task as set up by a teacher, and (3) the task as implemented by students. In this study, I categorized the tasks in participants' CMs and the tasks they planned according to the cognitive demand categories. I focused on the transition from the first task phase to the second: the changes teachers made to CM tasks to the way they planned to set up the tasks.

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Methods

Study Context and Participants

The purpose of the CT4EDU project is to support elementary teachers to incorporate CT into their mathematics and science teaching. CT is a broad set of thinking practices used by computer scientists (Yadav, Stephenson, & Hong, 2017). The CT4EDU project is focused on big ideas in CT, including *abstracting* important information from situations, *decomposition* of complex problems into simpler parts, and *debugging,* or finding and fixing errors. In a professional development workshop, participating teachers worked in groups to plan a math lesson, starting from CMs, that incorporated at least one of the CT practices ideas mentioned above. Two teachers from the project were chosen for inclusion in this study. Alice and Cindy (both pseudonyms) were using *Math Expressions* (Fuson, 2012), the CMs mandated by their district. Alice was a fourth-grade teacher with 15 years of experience. Cindy was a fifth-grade teacher with five years of experience.

Data and Analysis

I examined the *Math Expressions* lessons referenced by Alice and Cindy, as well as the tasks that were the focus of their conversations. I classified these tasks according to level of cognitive demand. Next, I used transcripts of Alice and Cindy's planning conversations to create descriptions of the tasks these teachers planned to pose to students. I classified these tasks according to cognitive demand. To understand how CT played a role in how teachers adapted the tasks, I read the transcripts of the planning conversations to identify decisions related to changes to the tasks. Next, I examined the explanations the teachers articulated for these decisions. I considered an explicit mention of a CT practice or reference to a CT handout as potential evidence of influence of CT on the teachers' reasoning. I coded the decision as influenced by CT when the teacher (1) related a decision to a CT practice's description, (2) described how a proposed change would provide opportunities for students engage in a CT practice, or (3) connected a decision to a CT practice as she reflected on the lesson-planning process.

Results

Alice

Alice was working from a *Math Expressions* lesson on estimation and mental math. Table 1 shows the initial tasks posed in the CMs and the task Alice set up in the classroom. Both tasks on the left can be classified as Procedures without Connections. Students are asked to produce an estimate and an exact total in the first task, but not to explain their reasoning. Students are asked to provide a solution method and a yes-or-no answer for the second task. The task on the right, by contrast, can be classified as Procedures with Connections. To engage with this task, students must think about the impact of estimating via rounding to the nearest hundred on the real-world context of the problem rather than merely do the rounding for themselves. Thus, the planned version of the task has a higher level of cognitive demand than the tasks as posed in the CMs.

Three of Alice's decisions were influenced by CT. First, Alice chose a task from the CMs to use as the main task in her lesson. She primarily attended to the two tasks at the left in Table 1, and decided to start with the latter because she wanted to give students an opportunity to *decompose* a problem. She felt the two-part format of the first task did the decomposition for students: "I feel like now, looking at this, this wouldn't be good because they're giving it to them. They're telling them how to break it down." Second, Alice changed the statement of the problem to prompt a discussion about different possible estimates and how those estimates differ from the exact total. Third and relatedly, Alice changed the numbers in the task. According to Alice, students always rounded to the highest place value—two-digit numbers to the nearest 10, three-digit numbers to the nearest 100, and so on. She expected students to use this rounding technique as they made estimates, and felt that changing the numbers to be in the hundreds would lead to estimates farther away from the exact total: "These numbers aren't gonna have them overestimate. So maybe change them so that the numbers are higher?" Estimates further away from the total, reasoned Alice, could lead to a discussion of *debugging*.

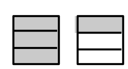
Table 1: Alice's Starting Tasks and Task as Set Up in the Classroom

Starting Tasks from *Math Expressions*	Task as Set Up by Alice
The best selling fruits at Joy's Fruit Shack are peaches and bananas. During one month Joy sold 397 peaches and 412 bananas. a) About how many peaches and bananas did she sell in all? b) Exactly how many did she sell? Tomas has $100. He wants to buy a $38 camera, a $49 CD player, and 2 CDs that are on sale 2 for $8. How can Tomas figure out if he has enough money for all four items? Does he have enough?	My friend gave me $930 to purchase items for a trip. The exact costs are $651 for his plane ticket, $112 for clothes, and $156 for meal gift cards. I rounded the amounts and added them to get an estimate of $1000. I told my friend he did not give me enough money, but he said I was wrong. I rounded the costs to the nearest hundred and added: 700 + 100 + 200 = $1000. Can you help me figure out what I did wrong? Did he give me enough? Did I round incorrectly?

Cindy

Cindy was working from a lesson on fractions greater than 1. Table 2 shows the tasks from the CMs and the task Cindy set up. The CM tasks can be classified as Procedures without Connections. Students can complete them by following the procedures given in the examples. The task on the right can be classified as Procedures with Connections. Students must think about the whole and provide two other representations of a fraction greater than 1, given one representation. Thus the cognitive demand of this task is higher than the tasks given in the CMs.

Table 2: Cindy's Starting Tasks and Task as Set Up in the Classroom

Starting Tasks from *Math Expressions*	Task as Set Up by Cindy
Change each mixed number to a fraction. **Example:** 2½ = 2 + ½ = 1 + 1 + ½ = 2/2 + 2/2 + ½ = 5/2 3 2/5 = ____ 2 3/8 = ____ (etc.)	Fill in the missing parts. In the unit fraction column, draw a ring around the whole. **Picture** **Sum of Unit Fractions** **Fraction** ¼ + ¼ + ¼ + ¼ + ¼
Change each fraction to a mixed number. **Example:** 13/4 = 4/4 + 4/4 + 4/4 + ¼ = 1 + 1 + 1 + ¼ = 3¼ 10/7 = ____ 12/5 = ____ (etc.)	 12/5 (additional rows were given)

Cindy made four decisions influenced by her attention to CT. First, Cindy decided to teach the *Math Expressions* lesson over two days to allow her to spend more time on representing fractions greater than 1. Cindy credited this decision to thinking about *decomposition*:

> The CT is helpful to me as the teacher, in a sense that I'm now looking through a finer lens at the lesson itself and thinking, gosh, the workbook does go in this order, this fast. But really breaking it down and trying to think like the students are, and really think about what challenges they have. And how I can decompose the lesson itself into smaller pieces.

Second, Cindy decided to launch the lesson by showing students one representation at a time (picture, or sum of unit fractions) and discussing how students could change one representation into the other. Third, Cindy incorporated pictures and sums of unit fractions into the student page so students' independent work would more closely mirror the class discussion. Fourth, she limited the examples to numbers less than 3, so drawing models and writing sums of unit fractions remained a viable strategy. Cindy related these three decisions to supporting students in realizing that symbolic representations of fractions are an *abstraction*:

> Yeah, that abstraction is heavy. Even having them consciously aware of what that abstraction feels like and looks like here. To talk and have that discussion when you go from the visual to the sum of unit fractions or to the mixed number and really highlighting that idea.

Discussion

Alice's attention to CT supported her in thinking deeply about how students would approach tasks. Thinking about decomposition led her to consider the impact of the CMs breaking problems into subparts for students—which is one way of lowering the cognitive demand of a task by changing a challenge into a nonproblem (Stein et al., 2000). Thinking about debugging led Alice to consider how she expected her students to approach rounding and the impact that approach may have in a real-world context. This suggests that thinking about CT practices supported Alice in making curriculum adaptations based on student thinking, which other studies have suggested leads to productive use of CMs (Choppin, 2011; Grant et al., 2009).

Cindy's attention to CT supported her in thinking about big mathematical ideas in her lesson. As she considered symbolic fractions as an abstraction, she began to consider the multiple ideas encapsulated in those representations (e.g., 7/5 is an abstraction intended to show that wholes are divided into 5 equal parts, we are considering 7 of parts, and so on). Cindy realized she did not think students would be able to "see" all this information in a symbolic fraction without more experience with other representations. This led her to decompose the lesson. Ergo, examining her CMs through the lens of CT helped Cindy unpack big mathematical ideas—another strategy research suggests leads to productive CM use (Stein & Kaufman, 2010).

This data does not allow me to empirically examine *why* the lens of CT practices led teachers to consider and adapt their CMs in this manner, but considering the conference theme suggests one possibility. While the CT practices highlighted here resemble disciplinary practices in mathematics, the import of these ideas from another disciplinary culture, computer science, may have aided teachers in engaging with them in ways that supported new kinds of pedagogical thinking. Decomposition, for example, is discussed in the Common Core State Standards (CCSSI, 2010), but only in reference to decomposing mathematical objects such as numbers or geometric shapes. Computer scientists tend to discuss decomposition of *problems* (Yadav et al., 2017). This broader nature of the object being decomposed seemed to support Alice in thinking about decomposing the steps of a problem (rather than a number) and to support Cindy in thinking about decomposing the multiple mathematical ideas in her lesson. As computer science education emerges as a unique research area, math education researchers may benefit from cross-disciplinary conversations that offer new perspectives on existing ideas.

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