ELEMENTARY TEACHERS' DISCOURSE ABOUT MATHEMATICAL REASONING

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Even though mathematical reasoning [MR] is at the heart of several elementary curricula around the world, very little is known about the meanings given to MR by teachers. In this paper, adopting a commognitive perspective (Sfard, 2008), we aim at better understanding the different meanings teachers give to MR. To do so, we used the Mathematical Discourse for Teaching framework (Cooper and Karsenty, 2018) to analyze elementary teachers' discourse about MR. Through individual and collective interviews, we gathered the data. We coded the data by first highlighting the vocabulary used to give meaning to mathematical reasoning by the teachers and, secondly, by identifying the utterances linked to their Mathematical Discourse for Teaching. Analyses revealed that elementary school teachers' discourse about MR is coherent with the prescribed curriculum.

Keywords: Teacher knowledge, Elementary School Education, Reasoning and Proof, Mathematical Knowledge for teaching.

This paper presents an analysis of teachers' discourse about mathematical reasoning [MR]. The MR is at the heart of several curricula around the world. In Quebec, where this project takes place, it is one of the three competencies of the elementary and secondary school curriculum (MEQ, 2001)[1]. According to Loong, Vale, Bragg and Herbert (2013), primary school teachers feel confused or uncertain about the task of defining Mr. Likewise, the meanings given to MR could play an important role in how teachers approach it in class (Stylianides and Ball, 2008). Taking a commognitive perspective, we aim at describing the discourse about MR of elementary teachers. In doing so, we want to better understand how MR can be fostered in classrooms from the teacher perspective.

What do we know about Mathematical Reasoning discourse at elementary level?

Despite a growing interest in MR and teachers' practices, very little is known about the meanings given to MR by teachers as well as how they promote its development in the classroom. Clarke, Clarke and Sullivan (2012) asked 104 elementary school teachers which MR related terms, from a given list, they frequently used in math class. Only four terms—explaining, justifying, proving and reasoning—were chosen by more than 50% of the teachers. To evaluate a professional development [PD] that aims at fostering MR in elementary classrooms, Herbert, Vale, Bragg, Loong and Widjaja (2015) explore the different meanings given to MR by teachers from Australia and Canada (Vancouver) involved in the PD. Their analysis highlights seven meaning categories that elementary teachers may attribute to MR: 1) thinking; 2) communicating; 3) solving problems; 4) validating thinking; 5) forming conjecture, 6) using logical arguments for validating conjectures; and, 7) connecting different mathematical aspects. Those categories emerged from the discourse developed during the PD. However, what about terms used and meaning given by teachers who never participate in this kind of PD? This study aims at investigating this question.

[1] The three competencies are 1) to solve a situational problem related to mathematics, 2) to reason using mathematical concepts and processes and 3) to communicate by using mathematical language.

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Mathematical Discourse for Teaching

From a commognitive perspective, cognition and communication are two aspects of the same ontology, i.e. discourse. Discourses are constituted of keywords, visual mediators, rules of discourse, routines and generally endorsed utterances (Sfard, 2008). Knowledge and practices are two aspects of the discourse associated with the teaching of mathematics (Cooper and Karsenty, 2018). So, in the same way, the investigation of the meaning that mathematical reasoning can take in elementary school can be done by considering the teachers' discourse, rooted in practice.

Cooper (2014) reformulated the Mathematical Knowledge for Teaching framework of Ball et al. (2008) from a commognitive perspective: Mathematical Discourse for Teaching [MDT]. As the MKT framework, the MDT framework is divided into two types of discourse: Mathematical discourse and pedagogical discourse. Mathematical discourse [MD] consists of common content discourse (the mathematical discourse that is common to a large portion of educated society), specialized content discourse (mathematical discourse that is typical of teachers of mathematics) and discourse at the mathematical horizon (patterns of mathematical communication that are appropriate in higher grade levels). Pedagogical content discourse [PCD] consists of discourse about content and teaching, discourse about content and students and discourse about the curriculum and resources.

Adopting this framework, we can reformulate our aims as: What are the keywords, visual mediators, rules, routines and generally endorsed utterances that constitute MDT of elementary teachers in relation to MR?

Some methodological insights

The data used in this paper came from a larger project that aims to document how MR is defined and fostered by elementary and secondary teachers. Six elementary teachers with 2 to 16 years of experience participated in one 60 minutes individual interview (Pseudonymes: Martine, Gisèle, Aurélie, Jeanne, Alice, Agathe). Five of them participated in a 120 minutes collective interview. All interviews were video or audio recorded.

Three different moments constituted the individual interview. First, the interviewer asked the participant to recall a moment of her teaching or to present a task that she gives her students in which MR would be promoted. This allowed us to stay in an area known to the participant. Furthermore, it informed about the learning environment that teachers considered favourable to the development of MR. Then, two examples of tasks including one with a student's solution were presented to the participant. The participant was then invited to decide on the possibility for a student solving these tasks to develop MR or not and to justify their answer. If the answer was positive, she was asked to describe the possible reasoning processes in their own words. Finally, to close the meeting, the participant was invited to give in a few sentences her definition of MR.

The collective interview sought to encourage exchanges between practitioners around MR so as to bring out the discursive elements shared by them. The first part of the interview aimed at defining MR. The interview therefore began with the question that had ended the individual interview: "How do you define mathematical reasoning in a few words or sentences?" This was followed by an activity where participants constructed a conceptual map with vocabulary words widely used to define MR during individual interviews or in the literature. The second part of the interview was to see how the teachers could reinvest the conceptual map to comment on students' written work. Finally, the group interview ended once again by offering each participant the opportunity to add something related to their definition of MR.

In order to analyze the data, the videos and audiotapes were viewed/listened to repeatedly, and transcribed (Powell, Franscisco & Maher, 2003). Using Nvivo software, a first layer of coding made it possible to highlight the keywords used to give meaning to MR by the teachers. A second layer of

coding made it possible to identify what seem as endorsed utterances linked to MDT for this group of teachers. By focusing mainly on keywords and endorsed utterances, discourse about the curriculum and resources [DCR] was particularly highlighted by the analysis.

Mathematical Discourse for Teaching and Mathematical Reasoning

When asked specifically to define MR, teachers use different keywords that seem to come from a common discourse about MR. For example, MR is linked to logic and argumentation, which are two words used to define reasoning in general dictionaries. But other keywords are more specific. For Martine, MR is to communicate and understand why. Similarly, Aurelie defines MR as explaining why. For Agathe it is also to communicate but to explain how. Gisele refers to MR as applying concepts and explaining what you have done. Alice used the metaphor of the toolbox. For her, MR is knowing when to use your toolbox, justifying and identifying and extrapolating patterns. Jeanne refers to organizing, thinking and making sense.

In relation to the PCD of the participants, generally endorsed utterances are usually embedded in DCR. Here are two illustrative examples.

Analyzing, making choices, applying, justifying and Mathematical Reasoning

In the teachers' discourse, analyzing, making choices, applying and justifying are important aspects of MR. Most teachers refer to those terms and they usually do so specifically by referring to the evaluation grid provided by school boards and based on MELS (2011) document.

> Aurelie: it's the evaluation criteria. 30 points for the analysis, 50 points for applying it … The last evaluation criterion is justifying, with 20 points.
>
> Gisele: Once I understand, I have analyzed the problem, then I have to make choices in what I know and what I think that will help me to reason with it.

This grid also *renders MR processes a linear structure* in the teachers' discourse as illustrated by Gisele's utterance above. Moreover, it is possible to draw a parallel between the grid and Pólya's problem-solving model: 1) understand the problem that is similar to analyzing; 2) develop a plan or make choices; 3) implement the plan or apply; and finally, 4) verify or justify. This is what Agathe feels in connection with MR:

> Agathe: Listen… I have the impression that reasoning with MR, well, this is the old one … this is the old problem solving from 15 or 20 years ago.

The criteria for assessing MR competency therefore play an important role in the discourse on MR. This role contributes to blurring the discourse on problem solving and MR.

Problem Solving and Mathematical Reasoning

In addition to being used as a quasi-synonym for MR, problem solving takes three other meanings for the teachers. First, it's a pedagogical method that can foster MR. Second, it's a competency evaluated with a particular type of task. Third, it's the type of task that evaluates problem solving. Those last two meanings are embedded as for both, problem solving is seen as more global and complex than MR.

> Alice: Well, that's why it's interesting to teach with problems too. So, not to make problems after the concepts, to bring the concepts with the problems.
>
> Martine: Well, the link I do between solving and reasoning is that… In fact, well, a situation to learn and evaluate, it should be complex. So, for sure, every child can have a different answer. Then, he [the child] uses the concepts uh that we include in reasoning [competency] because there is a need and then goes and solves it. So, I think reasoning is like a prior to solving [competency] because it is part of knowledge.

We can link those elements of discourse to the type of tasks used by teachers to evaluate each competency. Both types of task, namely situational problem and problem using application (MEQ, 2001), have different characteristics. The wording of the former is longer, includes a context, many steps to do, many concepts and processes to use and, as the students have to make choices in the data in order to solve the task, many solutions are possible (Lajoie & Bednarz, 2012). The wording of the latter is usually shorter, with one or two steps and the students have to choose the concepts and processes needed to solve it.

Discussion and Conclusion

Similarly to Herbert et al. (2015), the meanings given to MR by those 6 elementary teachers are broad and manifold. Likewise, as shown by the partial analysis presented, those meanings are tied to and somehow limited by teachers' DCR. In fact, the discourse found in the Quebec curriculum (MEQ, 2001), just like the one in math education literature, is also quite blurry (author, year). As MR is a competency which must be assessed with a specific grid and so-called *problem using application* task, this grid greatly colours their whole discourse. However, the evaluation criteria seem to favour a linear vision of the MR activity. Although questioned by the teachers, this linear vision of MR conveyed by the grid could limit learning opportunities for students as it gives the impression that MR is a series of steps to follow. Moreover, the criteria make it difficult for teachers to differentiate problem solving competency from MR otherwise than by the type of task used.

Like the discourse in the mathematics education community, the discourse of teachers is formed from a set of discourses related to different fields: educational institutions, psychology, pedagogy, mathematics education. These discourses are sometimes incommensurable. Thus, as they point out, enriching and clarifying the vocabulary related to the MR, whether in the curriculum or in training could, among other things, open up new possibilities for developing it in the classroom:

> Agathe: But that proves that we have to have a common language… We teach mathematics differently depending on our understanding.

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