CONDITIONAL PROBABILITY IN EARLY CHILDHOOD: A CASE STUDY

PROBABILIDAD CONDICIONAL EN LA INFANCIA INICIAL: UN ESTUDIO DE CASO

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This study investigated the intuitive knowledge of conditional probability in one four-year-old child. Six clinical interviews were video-recorded for analyzing transcripts and expressions from the child. Findings from this research suggest that the child has a pre-operational intuition about change in sample space in non-replacement situations. He also seems to have an intuitive understanding of independent events. Furthermore, although his judgments are mainly subjective, he does use quantitative justifications, although, inconsistently.

Keywords: Cognition, Early Childhood Education, Probability.

According to Nikiforidou, Pange and Chadjipadelis (2013) and Antonopoulos and Zacharos (2013), the study of probabilistic thinking in preschoolers is very limited. Jones, Langrall, Thorton and Mogill (1997) list sample space, probability of an event, probability comparisons and conditional probability as four key constructs in probability thinking; however, while the first three constructs have been investigated by several researchers, they point out that there are few studies related to conditional probability in young children. As one of the key constructs in probability thinking, it would be worthwhile to identify how this construct first begins to develop in young children. In this way, we can better understand how to advance children's early probabilistic reasoning. Thus, we examined the following research question: How does a preschooler think about conditional probability? In this study we investigated the intuitive knowledge of one four-year old child in situations involving conditional probability and independence. The study focused on these concepts from an informal perspective.

Literature Review/Framework

Xu and Tenenbaum (2005) demonstrated that preschoolers might possess some intuitive notions of conditional probability despite having not reached the formal operational stage identified by Piaget and Inhelder (1975). Jones and Tarr (1997) validated a framework for assessing probabilistic thinking through a characterization of four levels in conditional probability and independence. Following are the first two levels briefly described:

- Level 1. Subjective: Students' judgments are based on their own construction of reality, looking for non-existent patterns and imposing their own system of regularity. Students believe past outcomes will always affect the future outcomes, and they "deny the existence of independence" (Jones & Tarr, 1997, p. 51). The level of subjectivity does not allow any meaningful attention to independence and conditional probability.
- Level 2. Transitional: Students in this level occasionally use quantitative information for making conditional probability judgments.

Some researchers (Nikiforidou & Pange, 2010, Nikiforidou, et al. 2013) highlight the importance of considering intuition when investigating children's probabilistic thinking. In this way, Fischbein (1975) argued that children have intuitive probabilistic knowledge. These intuitive notions are implicit forms of cognition beyond cognitive structures and conceptual systems. Fischbein (1975) categorized intuitions into pre-operational, operational and post-operational. Specifically, pre-

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operational intuitions can be influenced by subjective or perceptual considerations, and operational intuitions are basic intuitions that are expressed through formal rules of logic.

Methods

In this study we used a case study analysis to investigate notions of conditional probability in one four-year-old child, Mack. Six clinical interviews of 5 minutes each were conducted and video-recorded. Tasks designed as games were used in each interview. In these games, Mack was asked to predict the outcomes in different scenarios. Correct predictions were reinforced with tokens. We analyzed each of the activities focusing on Mack´s predictions and his reasons for providing his answers. After transcribing each interview, in the video records we identified Mack´s expressions, actions, and fluency in his answers. From these records, we selected the video-segments that met one of the following criteria: showed any justification given by Mack, revealed specific indicators of a particular kind of intuition, or showed specific features associated with his notions of conditional probability. For the chosen segments, we included descriptions of Mack´s actions associated with his verbal expressions in the transcriptions. Later, we organized the completed transcriptions into two non-disjunctive groups: those that enabled us to infer Mack's intuitions, and those that reveal different kinds of judgments.

Results

We describe the results from the interviews according to Mack's intuitions and judgements. We begin with a discussion of the BALLS situation. In this situation, we mixed two green balls and two red balls in a bag. We asked Mack to predict the color of the ball that he thinks will have the greatest chance of being chosen from the bag. He answered "red," giving the reason that it is his favorite color. He then chose a red ball without looking inside. After that, he recognized that there were two green balls and one red ball remaining in the bag. Here, we suggest a pre-operational intuition associated with the chance of selecting a green ball, because after reviewing the bag, he quickly predicted the selection of a green ball for a follow-up selection. He gave as his reason for this prediction, that he likes all the colors (subjective consideration).

His pre-operational intuition of chance is further confirmed in the next trial in which there was one ball of each color left. In this case, he predicted that the red ball would be chosen, arguing that red is his favorite color. Similarly, in a scenario involving two red and two green balls in the bag, he selected a red ball which was then replaced; when he was asked which ball had the greatest chance of being selected, he answered red again arguing that it was his favorite color. Then, the red ball was not replaced, and he predicted that a green ball would be more likely to be chosen, again arguing that he likes all the colors. In this case, the number and color of the balls were not confirmed as in the previous situation. In a subsequent scenario, we added one green ball to the two red balls and two green balls. In this case, Mack predicted the green ball again arguing that he liked all the colors. In summary, when the chance of choosing the different colored balls was equal, he predicted a red ball arguing that red is his favorite color; but, when the chance of choosing a green ball was greater, he selected that color, arguing that he likes all of the colors. These situations reveal that Mack has an intuitive recognition that the sample space changes. For example, after selecting some of the balls from the bag, he changed his decisions according to the remaining number of colored balls without checking which balls were left in the bag. Furthermore, these decisions reflect an intuitive notion of conditional probability as the probability of the subsequent choice of ball appears to be conditioned on the knowledge of the previous choice.

We now turn to a discussion of Mack's judgements in situations involving independent events. In the COINS tasks, Mack was asked to predict the resulting color (yellow or red) after flipping a coin.

He was asked to first toss the coin without making a prediction. The outcome was yellow. On the next toss, he predicted the coin to be red. In the third toss, he looked more confident in his guessing than previously. Then, the following discussion took place:

Interviewer (I): Now, which color do you think that you will get if you throw the coin?
Mack (M): Mmm, red
I: Why do you think is red?
M: Because it's my favorite color.
I: Ok, it's your favorite color, very nice, let's see
M: [tossed the coin and got yellow]
I: Yellow! Now, let's do it again. What color do you think you will get?
M: Red
I: Red again? Ok, why red?
M: Dark

Mack's choices in the coin tossing situation seems to reveal an intuitive recognition that previous outcomes do not affect subsequent outcomes. In a follow-up scenario, we used two coins that were flipped one immediately after the other. Mack was asked to predict the color he would get tossing the second coin once he had tossed the first coin; he answered yellow, and the reason that he said was "shine!" In different scenarios, the main justification of his predictions continued being "my favorite color!" when his predictions were red. However, he seemed to change his justifications when he had a clearer intuition about the possible chance of the outcomes, as in the BALLS situation. He also included subjective arguments like "because is dark," "I am sweety," and "because it's my mom's favorite color."

In the FIGURES situation (Figure 1), Mack was asked to predict from which piece of cardboard the figure was chosen (situation inspired by Lucas, Bridgers, Griffiths, and Gopnik, 2014). Covering the pieces of cardboard, we selected a diamond and asked Mack from which piece of cardboard the image was taken. He linked the selection of the color (red cardboard) with the red flowers in the interviewer's blouse. Repeating the experiment twice, he changed the kind of judgment in the second repetition:

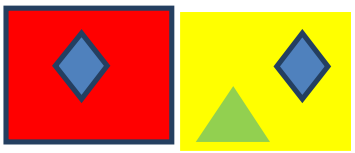

Figure 1. Pieces of colored cardboard

I: I take the blue diamond.
M: The yellow one.
I: Do you think it's from the yellow one?
M: (nodded)
I: Why?
M: Because there is more.
I: There is more what? There is more…
M: Shapes.

Although the quantity of shapes did not affect the chance, Mack´s attention moved away from subjective arguments to the quantity of shapes. In a similar situation in which there were two triangles on the yellow piece of cardboard and one on the red piece, Mack's first guess was that the triangle came from the yellow piece of cardboard, because "there is two of them." Although this was

not technically a prediction because Mack was looking when we selected the triangle, he seemed to be appealing to the number of triangles as a justification for his choice. In, yet, another situation where two triangles were on the yellow piece of cardboard and three on the red piece, his judgements associated with the number of triangles became clearer. We took one triangle without letting him see where it came from:

I: I took this triangle from one of these two cardboards, what cardboard do you think this triangle came from?
M: Red.
I: Why do you think is red.
M: Because there is more.
I: There is more in red?
M: More triangles.

However, in a following task, the triangle was not replaced, and he expressed the same argument in selecting the red piece of cardboard when they were the same number of triangles in each one.

Conclusions

This study identified the intuitive knowledge of a four-year-old child in situations involving conditional probability and independence. Although the child does not have any formal knowledge about sample space, he has a pre-operational intuition about changes in the sample space in non-replacement situations. In other words, Mack quickly adapted his choices and justifications depending on the different options he had once the sample space changed. Additionally, as seen in the COINS task, Mack did not seem to consider past outcomes when making future predictions. These findings seem counter to Jones and Tarr's (1997) framework related to the lack of attention to independence and conditional probability for children who are in the subjective level.

Mack's judgments were mainly subjective as Jones and Tarr (1997) characterize for the subjective level and Fischbein (1975) characterizes for pre-operational intuitions. However, over the course of the interviews, Mack changed his judgments from subjective to quantitative without instructional interventions. Although his quantitative reasoning was inconsistent and sometimes irrelevant, notions of quantity did influence his decisions. The trajectory of Mack's judgments provides an intriguing situation for further research. In particular, when he felt more confident about the possible answer, he appealed to the quantity of objects to justify his predictions. It seems that Mack has an intuition about the likelihood of an object being chosen that is associated with the quantity of objects involved. This suggests that intuitions about chance in conditional situations associated with quantitative reasoning may be stronger than Jones and Tarr (1997) argue for the subjective level. Nevertheless, although Mack mentioned number of objects in his justifications, he did not associate such numbers in his predictions.

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