LINKING A MATHEMATICIAN'S BELIEFS AND INSTRUCTION: A CASE STUDY

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Teachers' beliefs impact their instructional choices, but characterizations of that relationship are limited in college settings. Based on interviews and classroom video from three units of instruction, this paper examines a full-time instructor's stated beliefs about teaching and ways these beliefs manifested in their teaching. The instructor made curricular choices clearly aligned with their stated beliefs about math, learning, and teaching. Day-to-day instructional choices reflected these beliefs as well, but tensions between beliefs also manifested. Characterizations of the interactivity of classes are provided through descriptive and quantitative measures. These characterizations of instruction highlight changes in instruction throughout the semester.

Keywords: Classroom Discourse; Teacher Beliefs; University Mathematics

Beliefs impact the ways people perceive, interpret, and respond to situations (Pajares, 1992). Thus, numerous studies have examined teachers' beliefs, including three handbook chapters on teachers' beliefs in math education (Thompson, 1992; Richardson, 1996; Philipp, 2007). However, less is known about mathematicians' beliefs and their impact on instruction. Similarly, limited research has been conducted on semester-long college instructional practice. In response to these gaps, this study addresses the following research questions: (1) How did an instructor describe their beliefs about math, learning, and teaching? (2) How can their instructional practice be characterized? (3) What relationship exists between their beliefs and instructional practice?

Literature Review and Conceptual Framework

Extensive research describes the coordination of beliefs into a belief system. Philipp (2007) synthesized previous belief system characterizations as: "A metaphor for describing the manner in which one's beliefs are organized in a cluster, generally around a particular idea or object" (p. 259). Prior work on beliefs highlighted how they are influenced by a teacher's view of the nature of math (Ernest, 1991), prior school experiences, and immediate classroom situations (Raymond, 1997) as well as their effect on instructional practice (Wilkins, 2008). One of the distinctions between studies is how researchers address perceived inconsistencies in teachers' statements and actions. Early studies examined differences between what a teacher claimed and what they did (e.g. Cohen, 1990). Later studies examined both teachers' beliefs and practices before drawing conclusions (e.g. Schoenfeld, 2003; Speer, 2005; Speer 2008) and emphasized the importance of observing teachers for a long period to see how beliefs impact instruction (Skott, 2001).

While extensive research has been conducted on K-12 teachers' beliefs (e.g., Beswick, 2012), fewer studies have examined teachers' beliefs or instruction at the university level. Weber (2004) examined a real analysis professor's lecture-based teaching but observed the teaching style varied based on the material. Johnson, Caughman, Fredericks, and Gibson (2013) examined teachers' priorities for instruction while using Inquiry-Oriented (IO) materials, especially noting content coverage concerns, goals for student learning, and student opportunities to discover mathematics. Surveys of abstract algebra instructors have examined influences on lecturers' teaching (Johnson, Keller, & Fukawa-Connelly, 2018; Johnson, Keller, Peterson, & Fukawa-Connelly, 2019). Those most influential (in order of frequency) were their experience as a teacher, experience as a student, and talking to colleagues. These instructors self-reported their time spent on types of instruction, leaving questions about how to characterize college teaching.

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

The theoretical framework in this study is Leatham's (2006) construct of sensible systems. This framework posits that belief systems can be organized such that beliefs that seem contradictory to an outsider are not examined together by the teacher holding the belief, allowing "inconsistent" beliefs to coexist. Alternatively, certain beliefs could be held as ideal while others are given priority in specific situations. Generally, he suggested that if a researcher concluded a teacher's beliefs were inconsistent, the researcher did not have all of the information.

Methods

In this case study, the instructor participant, Dr. Bailey (a pseudonym), was a full-time instructor teaching an introductory abstract algebra course. Bailey's class met three times per week in 50-minute periods that were a mixture of lecture and "lab" days. They engaged in two semi-structured interviews (Fylan, 2005) lasting one hour each. Interviews were audio and video recorded and coded using thematic analysis (Braun & Clarke, 2006). Classroom data were collected in the middle of a unit on groups and through the whole units on group isomorphism and quotient groups. Classroom data were analyzed with the Toolkit for Assessing Mathematics Instruction–Observation Protocol (TAMI-OP) (Hayward, Laursen, & Westin, 2017) and the Inquiry-Oriented Instructional Measure (IOIM) (Kuster, Johnson, Rupnow, & Wilhelm, 2019).

The TAMI-OP is an observation protocol that aids recording what the instructor and students do in a classroom, broken into 2-minute segments of instruction. The IOIM was a rubric that provided a way to characterize how IO a class was. The IOIM uses a five-point scale and scores seven practices (below) that reflect the principles of IO instruction.

1. Teachers facilitate student engagement in meaningful tasks and mathematical activity related to an important mathematical point.
2. Teachers elicit student reasoning and contributions.
3. Teachers actively inquire into student thinking.
4. Teachers are responsive to student contributions, using student contributions to inform the lesson.
5. Teachers engage students in one another's reasoning.
6. Teachers guide and manage the mathematical agenda.
7. Teachers support formalizing of student ideas and contributions and introduce formal language and notation when appropriate. (Kuster et al., 2019)

Results

Instructor Beliefs

Bailey highlighted mathematicians' search for theorems as a purpose of math: "So I think mathematics is the search for theorems which…I would take to mean things that both can be proven…and then also the actual pursuit of proof…" Bailey emphasized actively doing math to learn it: "I'm a firm believer in learning by doing is best, so…every class I try to give the students something to do even if it's…here I'm gonna put this…example on the board for two minutes, let you guys work on it…." They based these ideas on how they learned: "I have to be …coming up with my own examples or coming up with my own proofs and just really synthesizing for it to stick." They were aware that how they learned could differ from how others learn, just as people have different ways of thinking in other contexts: "Different people have different frames for interpreting politics…so I think the same applies to learning."

Bailey discussed the role of different types of instruction within a class period when addressing the nature of teaching math. On lecture days, they would focus more on exploring the definitions and proofs in the class with a few smaller examples worked in. On lab days, they would expand the interaction that students were engaged in, especially for addressing examples.

They valued lecture as a way to make sure they taught all of the intended material and were satisfied with the interaction/coverage balance struck with two lectures and one lab per week.

Bailey identified two main ways that their beliefs about the nature of math, learning math, or teaching math were reflected in their instruction: the use of different types of instruction to reach different types of learners and an emphasis on students doing mathematics.

> It reflects my belief that people learn in different ways, and so, try not to use the same style throughout and also do different things….All my undergraduate mathematics classes were what I've been referring to as lecture.…I wasn't great at following what was going on in the lectures at that point in time. The group work is the kind of thing that would have helped me, so…putting in that different element for maybe people who do learn in a different way.

Their beliefs about creating a variety of learning opportunities for their students sprang from their experiences as a learner. In this case, the lack of alignment between their experiences and what would have helped them appeared to be formative. This relates to Johnson et al. (2018), in which the second most reported influence on instruction was experiences as a student.

Characterizing Instruction

Instruction is characterized based on data and analysis from the IOIM and TAMI-OP. IOIM practice scores are listed by practice (e.g. column P1 shows Practice 1 scores) with lecture scores on the left and lab scores on the right. TAMI-OP data rates are presented to the nearest whole percent. Counts of time blocks refer to numbers of 2-minute blocks (e.g. 9/31 segments lecturing means 9 of the 31 2-minute segments had some time spent on lecturing).

Table 1: IOIM Practice Scores in Lectures/Labs

Unit	P1	P2	P3	P4	P5	P6	P7
Groups	2/4	2/4	½	2/3	2/3	3/2	1/2
Isomorphism	2/3	2/2	2/2	2/3	1/2	3/2	2/2
Quotient Groups	2/2	2/2	2/2	1/2	1/1	3/3	1/1

In the Group unit, the lectures received low (1) to medium (3) IOIM scores, and the lab received medium-low (2) to medium-high (4) scores, as shown in line one of Table 1. These scores indicate the lecture days were not well aligned with IO instruction whereas the lab days were somewhat aligned with IO instruction. Similar scores were given in the Isomorphism unit. In the Quotient Group unit, the lab days received scores similar to lecture days; the only difference was on Practice 4, where the lab score was higher. Students engaged in less discussion with each other on lab days at all but one table, which depressed the IOIM scores. Across the three units, lecture scores held steady or decreased, except for Practice 7 in the Isomorphism unit. There, the lab started the unit, allowing some informal notation and ideas to come from the students before isomorphism was fully explained. The lab scores decreased or held steady except for Practice 6, where students were given more closure in a whole class setting in the last unit.

The results from the IOIM are also reflected in the TAMI-OP. In Table 2, we see lecture days in the Group and Isomorphism units were dominated by the instructor lecturing and included less time for students to work individually or in groups, whereas the allocation of time was flipped on the lab days. In the Quotient Group unit, more time was spent lecturing and students spent less time working than in previous units. Furthermore, unlike the previous units, where labs received a full day each time, this unit's labs received only partial days or spread over two days.

Table 2: Time Averages Across All Three Units

Day	Segments Lecturing	Segments Students Working	Segments Student Presenting	Segments Whole Class Discussion
Group Lecture	49/51	9/51	0/51	0/51
Group Lab	3/25	23/25	0/25	0/25
Group Ave.	68%	42%	0%	0%
Isomorphism Lecture	47/51	5/51	0/51	0/51
Isomorphism Lab	6/26	26/26	0/26	0/26
Isomorphism Ave.	69%	40%	0%	0%
Quotient Gp. Lecture	168/177	12/177	0/177	0/177
Quotient Gp. Lab	4/36	36/36	0/36	0/36
Quotient Group Ave.	81%	18%	0%	0%

Combining the information from the IOIM and the TAMI-OPs paints a picture of a class strongly guided by the instructor's mathematical knowledge but with some opportunities for student exploration. The mathematical authority rested with Bailey, who was in charge of moving the class forward. As the semester progressed, students were given less time to work and the amount of time the instructor spent lecturing increased, especially in the final unit.

Discussion and Conclusion

Dr. Bailey's stated beliefs about the nature of math focused on the structure of mathematics and the search for theorems. Their instruction reflected a belief in math as the search for theorems through their emphasis on proof in lecture, which they addressed by lecturing twice as much as they provided labs. Most of the time on lecture days was devoted to presenting proofs of theorems and thinking through implications of the work the instructor did at the board. However, the existence of two types of instructional days, opportunities to work on problems for extended periods, and opportunities to interact aligned with Bailey's stated desire to use many types of instruction to reach many types of learners. Although most groups experienced largely lecture and individual work time in class instead of varied amounts of discussion, this was still more instructional variety than might be expected in a "typical" lecture class. Bailey noted that their previous semester's section had been more interactive, so it is possible this was more due to the students' preferences than Bailey's intention. Here we have a tension between Bailey's belief that students should be interactive and that students should be free to make choices about how they want to learn. In keeping with Leatham (2006), it seems Bailey acted more on the latter belief, indicating they considered aligning to students' learning preferences more important than the incorporation of discussion while learning math.

Bailey seemed to intend to enact the interactive classroom described in the interviews. However, as the semester wore on, other factors seem to have gotten in the way. When behind their schedule, they pressed to finish by reducing the student work time to half days for labs. The instructor did not state a desire to reduce student work time, so it is possible they did not notice they were shifting how much time they spent on different activities. Nevertheless, this raises questions for further research on the influence of instructional pressures across a semester.

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