MAKING MEANING OF LEARNING TRAJECTORIES AMIDST MULTIPLE METAPHORS

Nicole L. Fonger
Syracuse University
nfonger@syr.edu

Amy Ellis
University of Georgia
aellis@uga.edu

In this theoretical report we focus on the issue of communicating learning trajectories (LTs) to researchers. There is great variation in the body of work on LTs including how researchers communicate what a LT entails, and the kinds of metaphors employed for making meaning of LTs. We elaborate possible affordances and limitations of different metaphors for LTs including "a garden path" and "growing flowers." This work has implications for how LTs are taken up by researchers, and also how LTs are leveraged to inform student-centered teaching practices.

Keywords: Learning Trajectories, Metaphors, Representations and Visualization

Learning trajectories are theoretical tools that elaborate transitions in students' processes of learning and goal-directed instructional supports. As a construct, learning trajectories can guide the design and study of teaching and learning by establishing predictions of how teaching and learning will unfold in tandem. Imagining, testing, and analyzing learning trajectories is core to the work of some design-based researchers, as are inquiries into how students learn mathematical ideas in response to instructional supports. Despite the integral nature of learning trajectories to some design-based research, the work of communicating such complex tools is challenging. Indeed, the domain of research on learning trajectories has grown and diversified in the approaches and theoretical orientations researchers take (e.g., see reviews by Empson, 2011; Lobato & Walters, 2017; and Fonger, Stephens et al. 2018).

This variation in theoretical orientation parallels what Simon (2009) articulated as a growth in the variety of theories of learning (and we might add, local instructional theories) being employed and developed in the field of mathematics education writ large. Amidst this growth, Simon recounts several challenges, including the need for researchers to engage in ongoing conversations to make theoretical choices transparent. In response to this challenge, we shed light on the practice of evoking visual metaphors for learning trajectories by considering the variation in both what learning trajectories communicate and how they are communicated in the research community.

Background and Major Issue

Variation in Learning Trajectories Research

One common definition for the construct of a learning trajectory (LT) is that it is a conceptual tool that links goals, instructional activities, and processes of students' learning (Simon, 1995). Learning trajectories vary widely in how they may (or may not) give evidence of students' mathematics with linked descriptions of the mathematical goals, learning activities, and/or the teacher moves that may engender such ways of understanding (cf. Lobato & Walters, 2017). As Empson (2011) elaborates, there has been a tendency for research reports in mathematics education to separate descriptions of learners' conceptions as evidenced on specific mathematical tasks, from the contexts, teaching, and tools, that might have engendered such learning (e.g., Steffe & Olive, 2010; Clements & Sarama, 2009). This focus on articulating students' conceptual trajectories on sets of tasks is a general trend in mathematics education research. Consider, for example, Hackenberg's (2014) research that details three epistemic algebra students' learning trajectories of fraction schemes and operations. In introducing these LTs, Hackenberg describes some ideas as to how the interviewer's questioning may have incited particular shifts in the students' reasoning on particular tasks. However, she

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acknowledges that many open questions remain regarding the role of instructional support in such shifts in students' learning processes.

Despite this apparent trend in how LT are taken up, not all LTs are communicated with this same orientation and emphasis. Some LTs more explicitly attend to things like teacher moves or other contextual features influencing student learning as a way to developing local instructional theory. Stephan and Akyuz's (2012) research, for example, leverages LTs as a tool to build an instructional theory. In their approach, instructional supports for students' understanding of addition and subtraction included classroom mathematical practices as engendered through tools, gestures, imagery, and taken as shared activities.

Evoking Metaphors as a Tool for Communicating Learning Trajectories Research

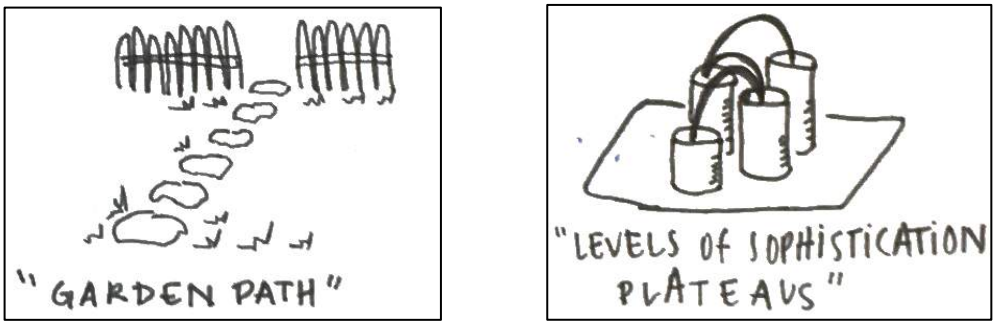
The challenge of conveying the simultaneity of change in students' learning and goal-directed instructional supports (as we believe learning trajectories as a research construct are primed to engender) may be a challenge of conceptual metaphor and related figural representations. Metaphors are essential to theory building and scientific inquiry (cf. Sfard, 1998). "Because metaphors bring with them certain well-defined expectations as to the possible features or target concepts, the choice of metaphor is a highly consequential decision. Different metaphors may lead to different ways of thinking and to different activities. We may say, therefore, that we live by the metaphors we use" (*ibid*., p. 5). Taking learning trajectories as the 'target concept' of our inquiry and theory building, we seek to understand the issue of communicating learning trajectories through an examination of the metaphors and meanings people make of them.

Different learning trajectories represent different things. Hence it is not always clear what types of information a particular learning trajectory might offer. Given this challenge, our inquiry into learning trajectories research is guided by the question of "What metaphors do researchers leverage in communicating learning trajectories?" We hypothesize that there is close link between what is communicated as a learning trajectory, and the conceptual and/or visual metaphor evoked in research.

Visual Metaphors for Learning Trajectories

Learning Trajectories Are Like "A Garden Path"

Sarama (2018) evoked the metaphor of a "garden path" to conceptualize learning trajectories, in which there are stepping stones that act as a developmental path to lead students through a gate—the goal state. From this view, learning trajectories must be interpreted by teachers and realized through social interaction around mathematical tasks. Indeed, the tight coupling of developmental progressions in children's thinking and sequences of tasks is evident in Sarama's research program (e.g., Clements & Sarama, 2014). Yet notice in Figure 1a, how the teacher, social interaction, and instructional activities are absent from the visual metaphor itself. Relatedly, Battista (2004) evoked the metaphor of "levels of sophistication plateaus" to characterize the "cognitive terrain" of learning processes for a learning trajectory (p. 186-187). In Battista's (2011) research, the sequence of tasks and ordered levels of sophistication in students' understanding are central to both what is conveyed in a learning trajectory and how it is communicated. Figure 1b offers a visual depiction of Battista's metaphor of "plateaus" of levels of understanding, which are complemented by narrative descriptions of related task type and instruction in his research.



(a) **(b)**

Figure 1. A visual depiction of (a) Sarama's (2018) "garden path" metaphor and (b) Battista's (2004) "levels of sophistication plateaus" metaphor for a learning trajectory.

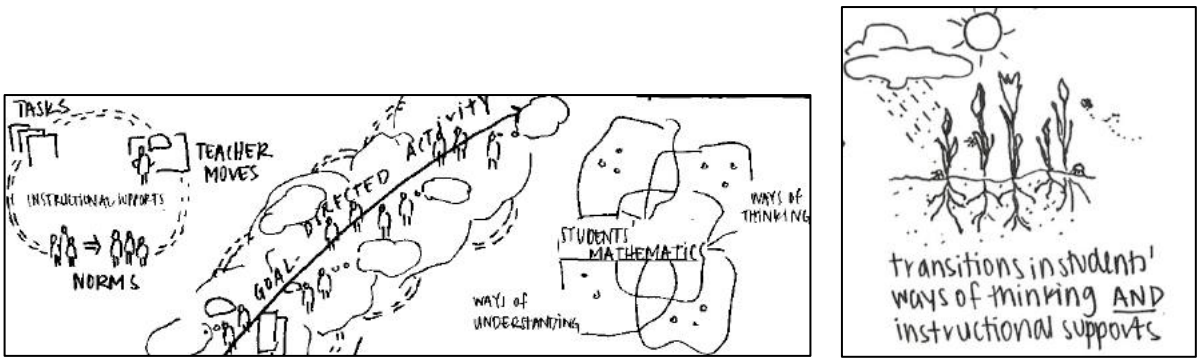
It is notable that the aforementioned visual metaphors (garden path and plateaus, cf. Figure 1) evoke a sense of capturing the shifts or changes in how students understand mathematical ideas as measured by student outcomes on mathematical tasks. In these "conceptual trajectories" (cf. Empson, 2011), the role of instructional supports is not necessarily captured in the metaphor itself. Said otherwise, these approaches to learning trajectories afford great insight into the nature of students' conceptions and mathematical reasoning that are possible given certain task situations, with possible descriptive connections to instructional supports. However, the nature, character, and nuances of the instructional intervention, teacher-student relationships, time, and place, are often masked in the levels and path metaphors for learning trajectories. As a field, advancing understanding of student cognition and related curricular supports (e.g., sequences of task progressions) remain important, valued research agendas. Indeed, recent research has indicated that teacher's knowledge of students is an important predictor for improving student learning outcomes (Hill and Chen, 2018). However, learning and instruction are complex inter-related processes, with learning as a function of teaching (cf. Empson, 2011). If a goal of learning trajectories research is to convey a progression from lesser to greater sophistication in students' mathematical learning processes toward desired goal states, these metaphors provide little guidance for how instruction might engender change in students' learning that goes beyond a coupling of knowledge of students and related mathematical tasks.

Learning Trajectories Are Like "Growing a Garden"

In some of our own work on learning trajectories (Ellis et al., 2016; Fonger, Ellis, & Dogan, 2019; Fonger, Ellis, & Dogan, forthcoming), we frame learning trajectories as a networked relationship between transitions in students' ways of thinking and related instructional supports. In Figure 2a we offer a visual depiction of how students' mathematics (their ways of thinking and ways of understanding, ala Harel, 2014), transitions together with instructional supports (tasks, teacher moves, and norms) as guided by goal-directed activity (arrow). In our work, this conceptualization of learning trajectories was supported by evoking the metaphor of "growing a garden." Depicted in Figure 2b, the growing a garden metaphor captures the complex interplays between how a plant grows in response to environmental conditions such as soil, sun, and water. Evoking this metaphor for a learning trajectory accounts for changes in students' conceptions of mathematical ideas (i.e., the growth of a flower) together with the nature of instructional supports including but not limited to teacher moves, task design features, norms, student discourse, and student activity with artifacts and tools (i.e., the environmental conditions).

With these depictions, we intend to capture a more nuanced model of both what change in students' conceptions might look like, and how the learning environment (of which tasks are just one part) supported such change. We argue for treating learning trajectories as more than "just a bunch of flowers;" learning trajectories convey the transitions in the growth of the flower in relation to the

supporting environment that evoked such change. This interpretation of a growing garden metaphor seems well-aligned to other research on learning trajectories (e.g., Stephan & Akyuz, 2012) that communicates the complexity of the interplay between the teaching and learning.



(a) **(b)**

Figure 2. Visualized Metaphors for Learning Trajectories as (a) a "growing garden", (b) Accentuating Transitions in Students' Mathematics and Instructional Supports

Discussion and Conclusion

To address the issue of what and how learning trajectories are communicated, we leveraged visual metaphors as a tool to make sense of two different approaches to learning trajectories research. In some approaches, learning trajectories are conceptualized as "a garden path", or "levels" wherein processes of students' learning and/or development as conceptual trajectories is foregrounded. Said otherwise, in a "garden path," characterizations of students' conceptual trajectories on sequenced sets of tasks is prominent. In another approach, learning trajectories are conceptualized as an interactive system of "growing a garden" wherein representations of goal-directed learning and instructional supports are taken together.

We invite conversation about the metaphors evoked in learning trajectories research as a way to address the issue of both what and how learning trajectories are communicated. We see a great need and opportunity to enrich the body of literature on learning trajectories (and learning progressions). In this report we argue that the use of interdisciplinary visual metaphors are productive for studying and representing learning trajectories. By articulating the metaphor(s) that guide our research, the products of learning trajectories research can become more explicit expressions of our theoretical assumptions about learning and teaching. Moreover, for researchers and practitioners concerned with learning trajectory based instruction (cf. Stzajn, Confrey, Wilson, Edgington, 2012), we hypothesize that by making theories of teaching (e.g., local instructional theory) more explicit in our communication of metaphors for learning trajectories, the field might improve the potential for learning trajectories to inform practice.

In close, by attending more seriously to the metaphors researchers evoke to communicate learning trajectories, the affordances and constraints of different approaches to learning trajectories becomes clearer. We intend for such elaborations to support the knitting together of a tapestry of research on learning trajectories that does not pit one approach against another, but that instead pushes for greater specification in the power of learning trajectories to advance research and inform practice as theoretical tools for research.

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