MATHEMATICS TEACHERS' PERCEPTION OF INDUCTIVE REASONING AND ITS TEACHING

PERCEPCIÓN DE PROFESORES DE MATEMÁTICAS DEL RAZONAMIENTO INDUCTIVO Y SU ENSEÑANZA

Landy Sosa Moguel
Autonomous University of Yucatan
smoguel@correo.uady.mx

Eddie Aparicio Landa
Autonomous University of Yucatan
alanda@correo.uady.mx

This paper presents the perception that middle school mathematics teachers attribute to inductive reasoning and its teaching from working with the concept of quadratic equation. The data was obtained from a questionnaire given to 16 teachers and from their expanded responses in a group conversation. Through the thematic analysis method, it was found that most teachers perceived this type of reasoning as a process to move from the particular to the general and as a way to guide teaching a concept through questioning. However, they encountered difficulties in using inductive processes to teach the concept and attach it to an inductive logic.

Keywords: Teacher Knowledge, Reasoning and Proof, Middle School Education.

Introduction

Inductive reasoning for learning mathematics in basic and middle school education is important for two reasons. On one hand, it is a means for the development of concepts and the resolution of mathematics problems (Molnár, Greiff, & Csapó, 2013; Papageorgiou, 2009; Sosa, Cabañas y Aparicio, 2019; Sriraman & Adrian, 2004) that assists the abstraction and generalization of the invariant characteristics of particular objects or situations. Klauer (1996) claims that it leads to "detecting regularities, be it classes of objects represented by generic concepts, be it common structures among different objects, or be it schemata enabling the learners to identify the same basic idea within various contexts" (p. 53). On the other hand, it supports processes to speculate, argue and generalize in mathematics (Cañadas et al., 2007; Cañadas, Castro and Castro, 2008; Conner et al., 2014; Martinez & Pedemonte, 2014).

This implies that middle school teachers should develop and interpret the inductive reasoning of students (AMTE, 2017; NCTM, 2000). NCTM (2000) establishes that this form of reasoning must progress in students throughout each grade and education level so that they can become more proficient in the formulation of conjectures and generalizations from specific cases. In this sense, it is desirable that teachers have clarity about inductive reasoning and the phases that go along with the transition from particular instances to the general. On the contrary, they may have difficulties incorporating it into their practice. Therefore, the goal of this study is to examine and describe the perception that middle school teachers show about the inductive reasoning in relation to the teaching of the quadratic equation concept.

Literature review

Much of the research on inductive reasoning and professional development of mathematics teachers has been conducted with preservice teachers and most of them focused on issues associated to the teacher cognition, such as ways of recognizing similarities by induction from numerical and figural representations (Rivera & Becker, 2003), levels of deepening understanding and strategies used to solve a generalization problem (Manfreda, Slapar, & Hodnik, 2012), the role of induction and abduction in making generalizations of classes of abstract objects (Rivera & Becker, 2007), and the relationship between inductive and deductive reasoning with learning styles (Arslan, Göcmencelebi, & Tapan, 2009). Results indicate that future teachers tend to induce numerically over strategies used

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based on the use of figures. Difficulties are also reported in generalizing quadratic patterns, even when a numerical pattern was identified. However, Sosa, Aparicio and Cabañas (2019) show that secondary school teachers who achieve generalization in these kinds of patterns are those who managed to connect inductive processes; they also identified difficulties to establish and abstract a pattern. This reinforces the need to study how teachers perceive inductive reasoning and its teaching.

Conceptual framework

In this study, inductive reasoning is understood as a means to produce generalizations from particular cases, be they ideas, qualities, objects, facts, phenomena or situations. This understanding is consistent with those who refer to it as a mental process oriented to infer laws or general conclusions through observation and connection of particular instances of a class of objects or situations (Glaser & Pellegrino, 1982; Haverty et al., 2000, Polya, 1957).

The works of Reid and Knipping (2010), Polya (1967) and Sosa et al. (2019) are examples of this understanding. Reid and Knipping (2010) identify three characteristics of inductive reasoning: it comes from specific cases to conclude general rules, uses what is known to conclude something unknown and, it is only likely but not true. Polya (1967) proposes the following four phases of such reasoning to discover properties, principles and general cases in mathematics: observing particular cases, formulating a conjecture, generalizing and verifying conjecture. More recently, from a cognitive approach, Sosa et al. (2019) report that the connection of the following three processes is necessary to achieve generalization inductively: observation of regularities, establishment of a pattern and formulation of a generalization.

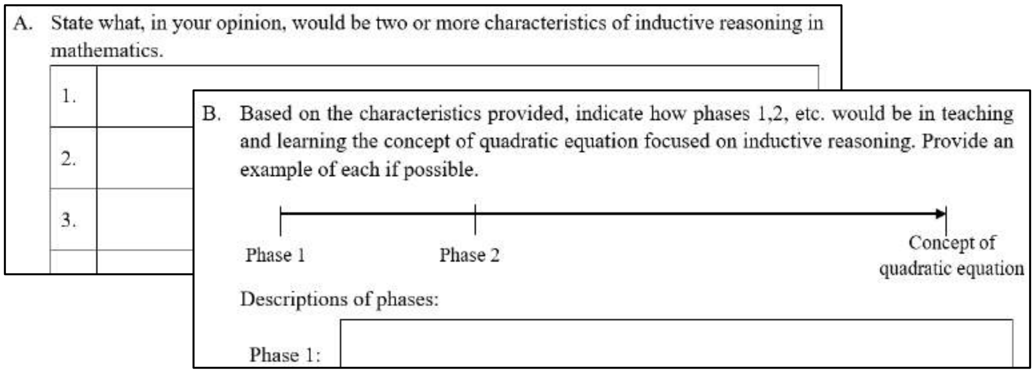
Methodology

Context and participants

This study is part of a professional teacher development program in mathematics, in which 16 secondary school teachers (10 women and 6 men) participated. The data was collected in the first of the five sessions that make up the program. Due to the relationship between inductive reasoning and generalization, as well as the difficulties of teachers to obtain a generalization of quadratic patterns as reported in the literature, the selection criteria for their participation was that they had at least one year of teaching experience in the third year of secondary school. This criterion is explained by the fact that, in the Mexican curriculum, "patterns and equations" is a topic associated with generalization, and the quadratic structure is studied in that education level.

Data collection

Data collection was conducted with a written questionnaire and audio recordings. The questionnaire had two items A and B (Figure 1). Item A asked for the enunciation of at least two characteristics of inductive reasoning in mathematics, and item B requested the description of the phases to be followed in order to teach some aspect of the quadratic equation in an inductive way. The replies were recorded in writing, and individually, and were subsequently communicated orally to the group for further information or clarification.



A. State what, in your opinion, would be two or more characteristics of inductive reasoning in mathematics.

1.
2.
3.

B. Based on the characteristics provided, indicate how phases 1,2, etc. would be in teaching and learning the concept of quadratic equation focused on inductive reasoning. Provide an example of each if possible.

Phase 1 — Phase 2 — Concept of quadratic equation

Descriptions of phases:

Phase 1:

Figure 1: Questionnaire for data collection

Data analysis

A thematic analysis was conducted to describe the perception of teachers considering the written and oral answers to item A. Then, the responses given to item B were associated to the categories of perception previously generated and contrasted with the conceptual framework in order to identify how teachers interpret inductive reasoning in teaching the concept of quadratic equation.

Thematic Analysis. This method consists of identifying, analyzing, organizing and systematically obtaining patterns (themes) in a data set by detecting and giving sense to the experiences and meanings shared in a group (Braun & Clarke, 2006; 2012). This helped to identify patterns of meanings in the common characteristics that teachers attribute to inductive reasoning and to form categories of their perception. To do this, the six phases of thematic analysis were followed: familiarize with the data, search for topics, review those that have potential, define and name themes, and produce a report (Braun & Clarke, 2012).

Results

Inductive reasoning perception categories

Five categories were identified on the perception of inductive reasoning, among them were *as a guide for mathematical knowledge* and *as a cognitive process.*

- **Category 1:** Inductive reasoning as a way to guide mathematical knowledge. This category consists in the fact that the students can be guided from their previous knowledge to new knowledge through questions. An example of this category is shown in the following excerpts of responses:

Teacher L: Give students an exercise and based on their previous knowledge draw their own knowledge. Create a brainstorm to learn what students know.

Teacher M: One of the characteristics is to begin asking key questions for the exercises and introducing students to the topic. Students begin to reason about the topic through questions and are able to visualize the previous knowledge. Guide questions. During the class, doubts may emerge [...] and questions may be asked [...], students can achieve the appropriation of concepts.

- **Category 2:** Inductive reasoning as a cognitive process. This category consists of perceiving it as a process to move from particular instances (ideas, particular cases or situations) to the inference of a general conclusion or result. For example:

Teacher E: It goes from the particular to the general...

Teacher N: It is a type of reasoning that consist of moving from particular to general ideas. Starting from concrete ideas to ideas in general. Generalize based on experiences of the given results.

Interpretation of inductive reasoning in teaching: logic and phases

Four different ways of interpreting teaching a concept based on inductive reasoning were identified. Eight teachers interpreted it as a guide for knowledge, such was the case of teacher M (Table 1). Five followed a deductive logic rather than inductive logic, for example teacher O; this means that they begin with the approach of general formulas or definitions of quadratic equations and conclude with a particular example. The deductive or inductive logic was not identified in the phases described by two teachers, they focused on "iconic" treatments based on the association of a quadratic property with the area of a square figure or the product of a number with itself. Strictly speaking, only the phases described by one teacher could be considered as an inductive logic. Overall, inductive processes were found to be absent in the phases proposed by the teachers for teaching quadratic equation, except for those described by teacher B (Table 1).

Table 1: Transcription of the phases proposed by two teachers

Phase	Teacher M	Teacher B
1	Previous knowledge: Introductory questions about algebraic expression, algebraic language, power, law of exponents.	Specific cases or situations which can be quantified, manipulated, or visualized are provided.
2	Application of the concept of "basic" shapes areas (with square shapes).	Different cases that meet the observed characteristic or property are asked.
3	Delete data and replace it with literals. Start with formulas.	It is required a prediction that this characteristic or property is fulfilled for other cases that are not tangible or directly observable.
4		A rule or formula that covers all possible cases is obtained; that is, a generalization.

Conclusions

Little clarity was identified in teachers about what inductive reasoning is. Most perceive or interpret it as a way of guiding mathematical knowledge in a teaching situation. However, this perception differs from the idea of inductive reasoning as a means for the construction of concepts; that is, to abstract and generalize the key characteristics of an object in specific situations (Sosa, Cabañas y Aparicio, 2019; Sriraman & Adrian, 2004; Klauer, 1996). It was also identified that few teachers perceive induction as a means to promote processes of generalization and resolution of problems. While reference is made to the transition from the particular to the general as a feature of this reasoning, the responses reveal a lack of clarity about the underlying processes because there is an inadequate interpretation when describing the phases to teach this mathematical concept; some of them even used a deductive logic. Therefore, it is necessary to compare and broaden teachers' knowledge of inductive reasoning through learning experiences in which they recognize and articulate inductive processes in contexts of mathematical generalization.

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