REFLECTIVE CONVERSATION AS A MEANS TO DEVELOP KNOWLEDGE IN FUTURE MATHEMATICS TEACHERS

CONVERSACIÓN REFLEXIVA COMO MEDIO PARA EL DESARROLLO DE CONOCIMIENTO EN FUTUROS PROFESORES DE MATEMÁTICAS

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This paper reports that the reflective conversation allows the development of professional knowledge in future mathematics teachers on the basis of questions about the nature and processes of construction of mathematical concepts, from both a mathematical and pedagogical point of view. The teacher of a didactics of mathematics course and ten future teachers participated in this study. The conversations held by them during three sixty-minute sessions were analyzed; the duration of these sessions was determined by the time it took to have the discussion of a question about even numbers and generalization. The type of knowledge developed consisted of recognizing the concept of even numbers as a process of mathematical generalization, andas an ability to be developed in elementary school students.

Keywords: Teacher Knowledge, Teacher Education.

This paper analyzes how reflective conversation helps to develop the knowledge for teaching the concept of mathematical generalization in future teachers. This interest is based on the fact that reflection and integration of content, and pedagogy are two central aspects of this development (Ponte & Chapman, 2016) equally, as is the collective (Arcavi, 2016; Chamoso, Cáceres & Azcárate, 2012; Horn & Little, 2010; Jaworski, 2006; Krainer & Llinares, 2010; Ponte, 2012; Preciado-Babb et al., 2015; Rasmussen, 2016; Rowland & Ruthven 2011; Santagata & Guarino, 2011).

It is now recognized that learning to teach requires the development of different types of knowledge in the teacher. In this sense, the problematization of the mathematical knowledge of the teacher has been the focus in several proposals in mathematics education in order to characterize and model it (e.g. Ball, Thames & Phelps, 2008; Carrillo-Yañez et al., 2018; Krauss, Neubrand, Blum, & Baumert, 2008; Kunter, et al., 2013; Pino-Fan, Godino, & Font, 2018; Shoenfeld, 2011; Llinares, 2012). In fact, a key question is "whether mathematical knowledge in teaching is located 'in the head' of the individual teacher or is somehow a social asset, meaningful only in the context of its application" (Rowland & Ruthven, 2011, p. 3).

On the other hand, little is known about the development of knowledge in future teachers (Thanheiser et al., 2014); in this regard it is stated that "it is important for programs to engage prospective teachers in learning opportunities that enable them to reconstruct their initial knowledge and understanding of mathematics teaching. This requires awareness and scrutiny of this prior knowledge. Reflection is a key process for achieving this" (Ponte & Chapman, 2016, p. 283). In this sense, we examine how knowledge of future teachers is developed in context of social interaction in the classroom, because it is a place that offers opportunities to teach, to learn from conversation and to reflect on their own knowledge and experiences (Horn & Little 2010; Kaminski, 2003; Toom, Husu, & Patrikainen, 2015).

One type of knowledge for teaching mathematics is generalization (Demonty, Vlassis, & Fagnant, 2018). Its importance can be seen mainly in two ways: as a way to teach for developing mathematical concepts (Davydov, 1990; Dörfler, 1991) and as an activity for the development of algebraic thinking

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in the study of patterns (Radford, 2014; Warren, Trigueros, & Ursini, 2016; Zazkis & Liljedahl; 2002). Therefore, mathematical generalization was the subject of conversation and reflection.

Conceptual Framework

The ideas of conversational learning of Pask (1976) and Kolb and Kolb (2017) were used and integrated to study how professional knowledge associated with mathematical generalization is developed, as shown in Figure 1. For these authors, learning comes from conversations on a topic that leads to the construction of new meanings and transforms collective experiences into knowledge.

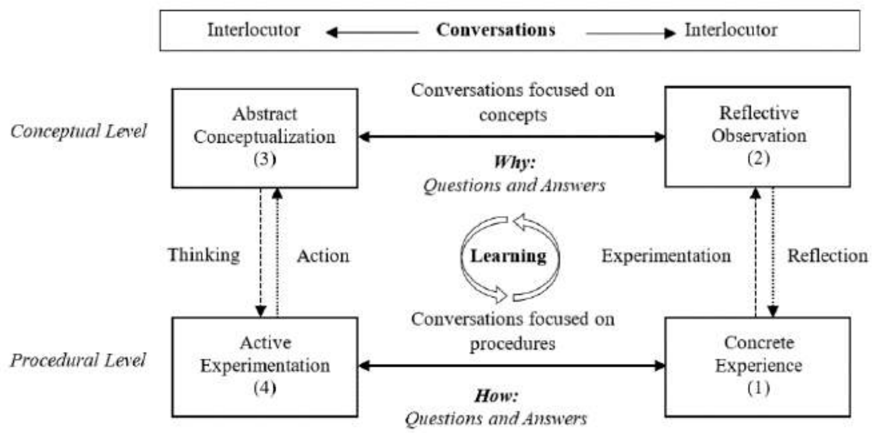

Figure 1: Reflective conversation and Learning based on Pask (1976) and Kolb and Kolb (2017)

We understand knowledge to be what is internalized in conversational interactions and it is relevant to those who share and participate in it (Gee, 2011). In this case, we refer to the specific knowledge of mathematics teachers with a clear orientation to the practical activity of teaching mathematics (Ponte, 2012).

Developing this knowledge from an RC implies the willingness to engage in a dialogue to negotiate meanings, accept questions and discuss the shared ideas of mathematics, its teaching and its learning (Earles, Parrott, & Knight, 2016). The knowledge of future teachers must be associated with their conversational context in such a way that the conversation establishes a space of participation to make explicit their ideas and knowledge related to a topic, making possible the emergence of meaningful relationships. Therefore, conversation allows the expression of thoughts in an open way, and by doing so, it gives way to a reflexive process that influences the meanings, thoughts, and actions of others; that is, it influences the development of knowledge.

Method

The development of knowledge of future teachers in the context of an RC was analyzed with a qualitative-interpretative methodology (Corbin & Strauss, 2008) considering interactions as the focus of the analysis (Kilpatrick, 1988). The analysis was carried out by organizing the data in (i) speaking shifts, and (ii) organizational sequence (Mazeland, 2006).

Seven women (W) and three men (M) from a training program for secondary and high school mathematics teachers at a public university in Mexico participated in this study. They were attending a didactic of mathematics course in their senior year. The participation of the teachers and the students was by invitation.

An open question was posed as a topic of discussion, since learning based on conversations can begin with questions associated to a topic (Pask, 1976) and the demands for solving a problem are a guiding factor for reflection (Dewey, 1938). The question was associated with the property of integer numbers and designed in such away that it could be answered by using the knowledge of arithmetic and of basic algebra and it allowed for the discussion of the idea of generalization, its teaching and its learning. The question was: *What can be said about the result of the multiplication of any two consecutive integer numbers?*

The analysis used the model shown in Figure 1 in the following way: First, the transition between the learning modes of the RC was analyzed based on the identification of the level of conversational interaction (procedural or conceptual levels). In the procedural level, it was considered that conversations are characterized by discussions focused on the construction or use of mathematical procedures, while the focus of the conceptual level is the use of conceptual ideas or theories that answer the posed question. Then, the experiences, reflections, thoughts and acts of the participants during their conversations were analyzed to characterize how it contributes to the development of their knowledge.

Results And Conclusions

Results show that the knowledge developed consisted in conceiving generalization as an important process to explore, explain and validate mathematical results and in recognizing its importance as a necessary mathematical ability to favor the teaching of arithmetic and basic algebra. Furthermore, the participants became aware that many mathematical concepts come from generalizations and raised the idea of the importance of using several representations (arithmetic, algebraic and geometric) to discover and test mathematical results, as happened in this case with the concept of even numbers:

M1: (…) I considered generalization as something of algebraic thinking, and the recognition of patterns and relationships between quantities proper of arithmetic. In this case, we can see a relationship between the quantities that increase from 2 to 4, then to 6, and 2 more units each time. It would be concluded that multiplying two consecutive numbers results in an even number, which is generalizing!

W4: I thought that there could be no generalization in arithmetic (...) but, after reflecting on what we understood as a generalization, as has been in this case: multiplying consecutive integer numbers results in an even number is a generalization; therefore, generalization can be made in arithmetic and algebra. In the sequence [2, 6, 12, 20, 30, …], number four is missing between 2 and 6, and the generalization that comes is that there are multiples of two. This is, an even number of the form $2k$.

RC fostered the development of knowledge to the extent that questions about the nature and construction of the concept of even numbers were made, from both a mathematical and a pedagogical point of view; it also promoted the free and open expression of ideas, answers and counter proposals associated with the initial question. This enhanced the transition between the four learning modes indicated in Figure 1. For example, participants moved from the concrete experience mode (CE) to the reflective observation (RO) mode by questioning and associating numerical relationships between algebraic structures. And they move from abstract conceptualization (AC) to the active experimentation (AE) mode by questioning meanings and looking for explanations and validations of their procedures. In this way, they managed to establish patterns and conceptualize an even number as a mathematical generalization (figure 2):

W2: How can I interpret the expression $2k$ in arithmetic and in algebra? I think that $2k$ can be seen as a whole area or it can represent an even number in arithmetic; in contrast, it may represent a quantity that is changing to double in algebra (...).

M1: Yes, let's say that by constructing a meaning for even numbers (…)

M2: (…) and considering it as the starting point for teaching even numbers.

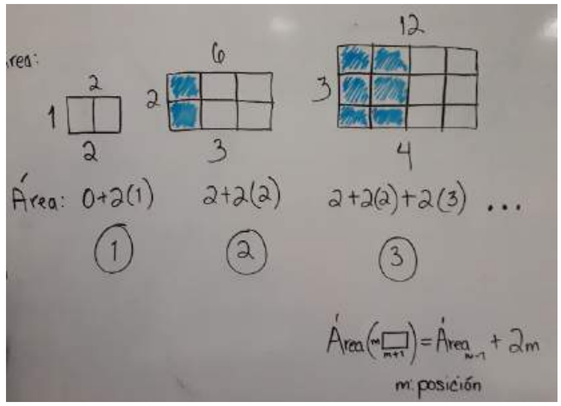

Figure 2: Consecutive multiplication, generalization and even number

We consider that the development of professional knowledge about mathematical generalization from an RC demands an exchange and articulation of ideas based on questioning the mathematical knowledge, its use and the argumentation of procedures and concepts used in the solution of problems, as well as the negotiation of meanings associated with them because this is what allowed the emergence of common understandings about the meaning of generalization and the cognitive demands for its teaching and learning.

These results are consistent with those reported by Demissie (2015), Jaworski (2006), Chamoso, Cáceres and Azcárate (2012) who found that participating in processes of collective inquiry promotes reflective thinking among peers. This study also confirms the results reported by Simoncini, Lasen and Rocco (2014) that a guided dialogue makes it possible for future teachers to obtain better perspectives of their teaching practices, including their thoughts and actions.

We plan to delve into how to incorporate RC in teacher training programs so that it can be a means to develop professional learning, promote reflections in relation to the professional practice, and encourage a shared vision of it (Preciado-Babb et al., 2015; Toom, Husu, & Patrikainen, 2015).

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