DOCUMENTING MATHEMATICAL LANGUAGE: DISTINCTION-MAKING AND REGISTER-FITTING

David Wagner
University of New Brunswick, Canada
dwagner@unb.ca

Karla Culligan
University of New Brunswick, Canada
kculliga@unb.ca

This paper advances theory for language use in mathematics learning contexts. The theory arises from a cross-sectional longitudinal study of student language use in Grades 3 to 11, both in English first-language contexts and French Immersion contexts. We point to translanguaging and the language-as-resource metaphor to consider the goals educators have for documenting students' mathematical language. We problematize deficit-oriented assessment of mathematical language and differentiate between using language for distinction-making and for register-fitting. Both are important. We introduce a tool for documenting language repertoires to recognize students' language strategies, including distinction-making and register-fitting.

Keywords: Classroom Discourse, Communication, Cross-cultural Studies, Probability

The understanding of any mathematics is mediated by language. There is a reciprocal relationship between language and conceptualization: language repertoires are necessary to convey an idea, and the language used shapes the way people conceptualize. This reciprocity led us to to identify children's language repertoires in a range of contexts. Here we present a tool for documenting language repertoire and we explain how it helps us think about theory.

The research data from which we draw examples comprised a cross-sectional longitudinal study in English-medium and French Immersion instructional contexts in an Anglophone region in Canada. We worked with students in Grades 3, 6, and 9 in the first year, Grades 4, 7, and 10 in the second year, and Grades 5, 8, and 11 in the third year.

Second language acquisition literature has shown that people are generally good at picking up and using the language strategies employed by others in interaction (Ellis, 1997; Long, 1985, 1996, 2007; Swain, 2000, 2008)—in other words, people are naturals at learning language. We claim that first language acquisition works similarly—people pick up and use the language strategies used by others. To listen for students' language strategies (as opposed to students' ability to understand and then use the strategies we exhibit), we deemed it necessary to design mathematical tasks that do not have us saying or writing language strategies we foresaw students using. We found it possible to avoid the specialist language of prediction in our tasks by constructing a narrative context for our questions.

We introduced the game of Skunk with a narrative like this in each classroom: "I was picking strawberries in the forest. When my basket was quite full, a skunk wandered into the berry patch. I ran away so the skunk would not spray me. And I lost the berries in my basket." Participants had a pile of beans (representing the berries), a cup (the basket), and a bowl (home). When the researcher rolled the die and called out the number, participants put that number of berries in their basket. A 6 represented the skunk. When it was rolled, everyone would lose the berries in their baskets. If they had "gone home" (dumping their beans into their bowl) before the appearance of the skunk, their berries were safe. We played seven rounds.

The day after playing the game in the classroom, we interviewed groups of students and played again but, instead of the die, we used six cards bearing the numbers one to six (the skunk). The interviewer would not replace the cards into the deck until the deck was completely played out, at which time it would be reshuffled. Thus the participants experienced the difference between independent and mutually exclusive events in probabilistic situations. During the card game, the interviewer would ask the participants to say why they made their choices about when to "go home." After the game, the interviewer would ask participants about specific things they had said earlier, asking for clarification on meaning. We had students work in pairs to encourage them to dialogue about their choices (e.g., "should we go home or stay in the berry patch?").

After transcribing the interviews, we collated students' language strategies for identifying certainty or uncertainty, organizing them into charts—one chart for each interview. The chart identifies each strategy and when it was used by referring to the turn number. A turn begins and ends with a change in who is speaking. The structure of the chart emphasizes who is the first person to use each language strategy and who uses it after that. We show one such chart here (Table 1), from an interview with four English-medium Grade 6 students. The strategies are presented in the order that they are used. For example, the first language strategy for uncertainty was Bal's use of 'probably' in turn 21. Bal used 'probably' again in turns 40, and 149. (Names are pseudonyms. "Int." refers to the Interviewer.)

Table 1: Grade 6 English group – expressions of certainty and uncertainty

Certainty	Certainty (continued)	Uncertainty
Simple Assertion		'probably'
1st user: Col (23, 44, 60)	…	1st user: Bal (21, 40, 149)
2nd user: Int. (49)		'it depends'
3rd user: Bal (62, 122,	'got to' / 'gotta'	1st user: Daz (22)
129)	1st user: Adi (260, 265)	2nd user: Adi (33)
'have to'	2nd user: Int. (262, 289)	'usually'
1st user: Int. (49, 81, 255,	3rd user: Daz (283)	1st user: Bal (25)
262, 279, 295, 297, 299)	4th user: Adi (284)	'you never know'
'sure'	'you know it's got to'	1st user: Daz (35)
1st user: Researcher (121)	1st user: Col (261)	2nd user: Bal (238)
'you know'	'has to'	3rd user: Int. (239)
1st user: Daz (197)	1st user: Adi (265)	'could'
2nd user: Int. (361)	'a rule'	1st user: Daz (35)
'probably'	1st user: Col (290)	2nd user: Int. (303, 305, 311,
1st user: Daz (197)	2nd user: Daz (291)	313, 317)
'I know'	3rd user: Int. (293, 295, 357)	'I/you think'
1st user: Col (226)	'can't'	1st user: Int. (30, 66, 69, 126,
'need to'	1st user: Int. (327, 329, 336,	128, 141)
1st user: Col (228, 294)	352, 361, 363)	2nd user: Col (41, 106)
2nd user: Daz (282)	'not allowed'	3rd user: Bal (105, 125)
'you know you're going to'	1st user: Bal (337)	'not sure'
1st user: Bal (238)	2nd user: Int. (338, 354, 361)	1st user: Int. (45, 47)
'you never know'	'impossible'	'I don't know'
1st user: Bal (238)	1st user: Col (340)	1st user: Daz (227)
…	2nd user: Int. (341, 344, 354,	'a chance'
	361)	1st user: Bal (304)
		2nd user: Int. (305)

Problematizing deficit assessments of mathematical language

We acknowledge that we found it hard to avoid deficit assessment even though it was our expressed intention to avoid it. For example, we expected students to use modal verbs to make distinctions in degrees of certainty—as we had found in earlier work (e.g., Wagner, Dicks & Kristmanson, 2015)—

ranging from negative root modality (e.g., 'it is not six') to positive root modality (e.g., 'it is six') and different levels of modulation in between.

However, we noticed that Adi, Bal, Col and Daz here did not use some common modal verbs. For example, after giving the students a chance to use the modal verb 'can't' on their own, the interviewer used it multiple times (starting in turn 327), even explicitly asking the students what it meant. But the students did not use it. It was clear that they understood it, because when asked about it they made distinctions between things that are 'not allowed' (Bal in turn 337) and things that are 'impossible' (Col in turn 340). If we were to rate their language use on a checklist, how would we assess their use of the word 'can't'? We can say they did not use the word. But it would be inappropriate to say they do not have the word in their repertoires. The fact that they did not use it does not mean they cannot use it. To illustrate further, we consider the word 'impossible'. If we had ended the interview a little sooner, before turn 340, we would not have known that it was in Col's repertoire. We can say that at least Bal and Col understand 'can't' because they responded well to questions using the word. They even demonstrated sophisticated understanding by making distinctions between impossibility due to logic and due to authority. Rowland (2000) has documented the language of this distinction.

Further, we see that Bal and Col responded with understanding to the word 'can't'. What can be said about the others in the group? We argue that it would be inappropriate to say that Adi and Daz did not understand 'can't'. While they did not use the word nor respond directly to the word being used, there was no reason for them to speak about it because Bal and Col already did so. We entered the research project with a principled decision to avoid deficit assessment. We found it difficult to avoid deficit approaches in our read of the data. Ultimately, our data gave us evidence to reject deficit assessments of language.

Translanguaging for distinction-making and register-fitting

Distinction is a word that appears multiple times in our theorizing above. Our stance of seeing language as resource (Martínez, 2017; Moschkovich, 2007, 2013; Planas & Setati-Phakeng, 2014; Ruiz, 1984) led us to appreciate the language work done by the students in our data across the ages. This led us to ask what goals we would promote for mathematics educators in relation to mathematical language. We settled on these three: (1) understanding mathematical concepts, (2) ability to use language to make mathematical distinctions, and (3) ability to sound knowledgeable (fitting the genre, the grammar, the lexicon)

We assume that all mathematics educators are interested in supporting students to develop understanding of mathematical concepts. With an interest in mathematical language, it is common to say that students should also be able to communicate their mathematics. This goal compels us to ask what it means to communicate mathematics. We differentiate between successful communication of an idea, which we call *distinction-making*, and using 'correct' language, which we call *register-fitting*. As shown by the students Adi, Bal, Col and Daz, and by the students in every other interview in our research from Grades 3 to 11, it is possible to communicate conceptual distinctions without using conventional language.

We claim that communicating mathematics successfully means being able to make mathematical distinctions in a way that others understand. We use the new theory of *translanguaging* here, introduced by García and Wei (2014), to challenge the neat boundaries people often imagine around languages. We aim to appreciate the range of language strategies people use, no matter how they cross lines of recognized languages (e.g., English or French), and variations within languages (mathematics registers, dialects, etc.).

For example, in relation to prediction, it is important to have language strategies that distinguish between certainty and uncertainty. This can be done with adverbs like 'certainly' and 'possibly.' Bal used adverbs—'probably' thrice, and 'usually' once. The distinction can also be made with adjectives, such as 'impossible' which was used by Col first in this interview. The distinction can be made with modal verbs as noted above and with a distinction between knowing and thinking, also noted above.

Further sophistication is possible with distinctions between levels of certainty (modulated certainty)—for example, Bal's adverbs 'probably' versus 'usually.' Other than that distinction we did not find modulated certainty in this group. For further examples we can point to data from a group of three Grade 9 French Immersion students. (We do not show the table due to space restrictions.) To identify a higher probability, Enk said 'plus de chance' (more chance) early in the interview (turn 14). Much later, Gyl said 'une bonne chance' (a good chance) (turn 127). To identify lower probability, Gyl said 'un sur trois chance' (one in three chance) early in the interview (turn 20) and 'une petite chance' (a small chance) later (turn 90).

In addition to making distinctions clearly, we have seen that mathematics teachers value students using 'proper' words 'properly'. We use the word *proper* in quotes because it can only refer to loosely defined expectations for standard lexicon ('proper' words) and standard grammar (words used 'properly'). This means that educators want to induct students into a community of mathematicians, who, presumably, use the words and grammar of the mathematics register—the specialized methods of communication used amongst the mathematically literate (Barwell, 2007; Halliday, 1974; Pimm, 2007). We see this *register-fitting* as different from distinction-making. However, there is a connection: as people move to using more conventional language it becomes easier for others to understand their meaning. For example, if we use language that you know and use, you will more likely understand us. This is the unitary force of language in Bakhtin's (1981) metaphor—the centripetal force (Barwell, 2014).

French Immersion contexts are especially interesting to us in terms of register-fitting because there are two explicit goals: learning mathematics and learning French. We suggest similar goals in first-language medium classrooms too: learning mathematics and learning to communicate mathematics, which would include both distinction-making and register-fitting.

Two of the four strategies used by students to modulate certainty in the French group used improper French: 'un sur trois chance' (turn 20), and 'une petite chance' (turn 90). We could criticize the students for using improper French (a deficit assessment). Alternatively, we could honor them for using the language strategies available to them to make the distinctions they intended to make. Strong language speakers are inventive with language to communicate their ideas. There are plenty of other examples of 'improper' language being used powerfully across the corpus of data in both French and English interviews.

But deficit views of 'proper' language are even more complicated. We consider the Francophone interviewer, who said 'c'est peut-être pas' ('it might not be') in turn 98. This too was successful communication in non-standard French (the speaker dropped the 'ne' in front of the verb 'est'). However, in oral French it would be weird to say the standard French 'ce n'est peut-être pas.' The interviewer was using standard oral French. We believe that anyone who has transcribed natural language data will have realized that people do not often speak in proper sentences.

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