PROFILING THE USE OF PUBLIC RECORDS OF STUDENTS' MATHEMATICAL THINKING IN 4TH-8TH MATHEMATICS CLASSROOMS

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Centering class discussions around student mathematical thinking has been identified as one of the critical components of teaching that engages students in justifying and generalizing. This report shares analysis from a larger project aimed at describing and quantifying student and teacher components of productive classrooms at a fine-grain level. We share results from this analysis of 39 mathematics lessons with a focus working with public records of students' mathematical thinking.

Keywords: Instructional activities and practices, Teacher Education-Inservice/Professional Development

The goal of *Working with Public Records of Students' Mathematical Thinking* is to make student thinking available to all students (Ghousseini, 2009), and to maintain common ground (Staples, 2007). This may look like recording student ideas (Cengiz et al., 2011; Staples, 2007) and engaging the class to work with it. Publicizing student work has the potential to position students as contributors to mathematics (Cohen, 1994). We illustrate an analysis of how public records of students' thinking were used in 39 lessons of grades 4-8 classrooms to productively generate meaningful student discourse. Students can learn mathematics when engaging with each other around mathematics (Schwartz, Black, & Strange, 1991). Teacher prompts that elicit reflection, communication, and meaningful explanations regarding a student's work and their thinking have been identified as essential and beneficial for mathematical learning and understanding (Hiebert, et al., 1997; Henningsen & Stein, 1997; Hiebert & Wearne, 1993; Kazemi & Stipek, 2001).We hypothesized that lessons in which teachers engaged students in examining public records of students' mathematical thinking would generate more and higher-levels of student discourse. Our research questions were: (1) How prevalent are public records of students' mathematical thinking within the lessons? (2) Do lessons that contain public records include more student-level engagement, specifically higher-level cognitive engagement? Do those lessons that also contain selected and sequenced public records include even more than those with either a) no selected and sequence public records and b) more than those with no public records at all?

Theoretical Orientation and Analytic Framework

There is a general consensus in the mathematics education community that high-quality mathematics classrooms are those in which student voices are heard, and student thinking is leveraged as the means to move instruction forward (e.g., Ball, 1993; Jacobs & Spangler, 2017; Nasir, & Cobb, 2006; Schoenfeld, 2011; Turner, Dominguez, Maldonado, & Empson, 2013). Enacting practices that foreground student thinking is complex, requires intentional and strategic moves, and persistence in enacting these moves over time (Staples, 2007; Boaler & Staples 2008; Franke, Kazemi & Battey, 2007). Mathematically productive teaching routines are a set of teaching routines designed for accessing and working with student mathematical thinking. Research

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has emphasized attending to students' mathematical thinking as one of the most essential aspects of impactful teaching (Jacobs & Spangler, 2018; Lampert et al., 2013).

One such teaching routine is *Working With Public Records of Students' Mathematical Thinking* (described above). This routine can be situated within the teaching routine *Working With Selected and Sequenced Student Math Ideas*. The goal of this routine is to advance student understanding by fostering connections related to the core mathematical ideas on which the lesson/task focuses. Once teachers have learned about how their students are thinking, they need to choose how to build ideas with the whole class by selecting and sequencing how student ideas are shared (Stein, Engle, Smith, & Hughes, 2008, Stein & Smith, 2011).

Methods

The 39 coded lessons for this project stem from two urban school districts in the United States: grades 4-5 were from a large urban district, whilst grades 6-8 came from a mid-sized urban district. The 20 lessons from the middle school were taken from each teacher at the end of the school year, and the 19 lessons from the elementary school teachers were a stratified random sample, according to Mathematical Quality of Instruction (Hill, 2014) scores. Because this paper focuses on two teaching routines (*Working With Public Records of Students' Mathematical Thinking* and *Working With Selected and Sequenced Student Math Ideas*), all lessons were coded for those two teaching routines. Each lesson was also coded for Students' Habits of Mind (HoM) and Habits of Interaction (HoI). HoI focus on students' verbal interaction with the teacher as well as with one another. HoI include *Explaining* their thinking, asking *Genuine Questions*, *Revoicing* other students' contributions, *Private Reasoning Time*, *Compare* logic and ideas for similarities or differences, exploring multiple *Pathways* to solving a problem, and *Critique* one another's ideas. HoM can happen within an HoI and focus on the cognitive activity embedded within their verbal interaction. HoM are noted here as *Representations (Reps), Connections* within and across two mathematical concepts, strategies, or structures, *Regularity and Structure* using patterns, properties, or mathematical structures, *Metacognition (Meta)* or reflection on their own thinking, recognizing, examining, or using their own or each other's *Mistakes,* engage in *Meaning of tasks and terms, Justify* their thinking, and *Generalize* ideas. To summarize, HoI are the ways a student can interact with others whereas HoM are the mathematical activities embedded within such an interaction. These codes were developed for a larger study involving the Math Habits Tool, which was developed to capture *mathematically productive components* of classrooms in terms of both student and teacher in-the-moment actions.

All coding was completed by graduate students who took part in a three-day coding training camp that focused on the various student and teacher-level codes used in this project. Each lesson was then assigned to two graduate students to code independently. After each coder had completed their initial coding of the lesson, the pairs of coders meet to compare their independent coding and reconcile any differences and disagreements. Disagreements that could not be reconciled between the two coders were sent to a third person for final decision.

In considering our research questions, we grouped the 39 lessons into three themes: (1) lessons containing public records where at least two public records were selected and sequenced; (2) lessons containing at least one public record, but none that were selected and sequenced; and (3) lessons containing no public records. We then compared those groups in terms of quantity and type of HoI, and HoM within the lesson.

Results

Of the 39 lessons, 26 lessons (67%) did not contain public records of students' mathematical thinking (Group 3), thus, student work was not displayed and worked with at all. Of the 13 lessons (33%) that did include public records of students' mathematical thinking, six were further situated in a *selecting and sequencing* routine (Group 1), while seven were not (Group 2). Thus, only 33% of the lessons contained student work that was actively displayed and worked with, and about half of those were situated in a selected and sequenced routine.

Across all three groups, the Habits of Interaction *Explain* and *Questions* were frequently used. Similarly, the Habit of Interaction *Private Reasoning* occurred sporadically throughout some of the lessons. Thus, explaining mathematics, asking genuine questions, and prompting students to use private reasoning about mathematics are habits of interaction that are seemingly not dependent on reasoning with students' work within a public record, so we removed those three HoI from our next level of data analysis. *Generalize* was not present in any of the lessons, so it too was removed from the next level of data analysis.

We found that while there was generally infrequent use of higher-level Habits of Mind and Interaction across all 39 lessons, lessons that did use public records engaged students in higher-level HoM and HoI more frequently than lessons that did not. Furthermore, lessons that selected and sequenced the public records were found to include student engagement in these codes more often than lessons that did not select and sequence their public records. (See table 1.)

Table 1: Percentage of Lessons in Each Category with Relevant Student Habit Occurrence

	Compare	Pathways	Revoice	Critique	Reps	Connect	Structure	Mistakes	Meta	Meaning	Justify
Group 1	83%	67%	83%	50%	83%	67%	33%	50%	50%	17%	50%
Group 2	14%	0%	29%	14%	86%	29%	43%	14%	29%	0%	0%
Group 3	8%	4%	0%	0%	23%	12%	8%	4%	0%	12%	8%

Moreover, we found this to not only be true of the lessons, but within the public records themselves. Next, we consider how the student engagement within a lesson compares to the engagement specifically during a public record portion of class. Table 2 highlights the average percentage frequency of a student habit for a whole lesson in the group's top row, and the average percentage frequency of a student habit for the public records portion of a lesson in the group's bottom row. For example, of all the higher-level student habits used in Group 1 lessons, 15% were *Compare* and 8% of those habits took place within a public record. Because Group 3 lessons contained no public record, there are no student habits within a public record to display (i.e. the second row is empty). Notice that 50% or more of the student habits in Group 1 lessons happen within a public record, and with the exception of *Compare*, *Critique*, and *Mistakes*.

Table 2: Frequency Percentage of Habits Per Lessons & Public Record in Each Group

	Average Time	Compare	Pathways	Revoice	Critique	Reps	Connect	Structure	Mistakes	Meta	Meaning	Justify
Group 1	56:50	15%	7%	14%	4%	28%	12%	2%	3%	8%	2%	5%
	17:46	8%	4%	10%	2%	22%	8%	1%	2%	7%	2%	5%
Group 2	58:30	2%	0%	14%	5%	43%	17%	10%	2%	7%	0%	0%
	09:02	0%	0%	12%	2%	26%	14%	7%	0%	5%	0%	0%
Group 3	45:00	11%	9%	0%	0%	32%	7%	20%	2%	0%	7%	11%

Discussion and Future Research Plans

Only 13 lessons (33%) engaged students using a public record of students' mathematical thinking. Only 6 (15%) lessons selected and sequenced the public records. Lessons with public records showed a higher percentage usage of higher-level cognitive engagement. Lessons which selected and sequenced the public records engage student in higher-level mathematical habits consistently more than lessons that did not. In fact, on average, lessons that selected and sequenced the public records of students' mathematical thinking showed a 46% increase in higher-level cognitive engagement compared to lessons that did not.

One explanation for this drastic difference is that in selecting and sequencing public records, students are exploring multiple pathways, comparing strategies, and inevitably critiquing and debating any contradictory or different ideas. Thus, by selecting and sequencing students' ideas, teachers make these habits of interaction more accessible for the students and can more advantageously create a dialog around multiple ideas.

Close to 50% or more of the student codes in Groups 1 and 2 lessons occurred within public records. Thus, public records are creating a time for students to engage in mathematical discourse more frequently and at a higher-level than time outside of the public record.

Although important to make student thinking available to all students and work with it, it is not enough. Providing access to students' ways of thinking offers ways of engaging; however, without selecting and sequencing the engagement is shallow and less frequent. Thus, by selecting and sequencing the public records of students' mathematical thinking, an exploration and dialog using the habits of interaction can be sparked to ignite the higher-level conversation that leads to deeper, more frequent usage of habits of mind such as making meaning and justification. Therefore, as evident from literature (Stein, Engle, Smith, & Hughes, 2008, Stein & Smith, 2011), having students' present their ideas to the class is not as effective in creating productive student discourse as carefully monitoring, selecting, and sequencing student ideas. Moreover, the results we have stated here illustrate the effects on students' engagement when a teacher effectively selects and sequences students' mathematical ideas. Further work will involve continued analysis of 61 more lessons to see if this pattern still holds. Additionally, this work focused on only student engagement in the lesson, but future work will also include analysis on teacher prompts for student engagement.

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