SUPPORTING FRACTIONS AS MEASURES IN AN ONLINE MATHEMATICS METHODS COURSE

Diana Moss
University of Nevada, Reno
dmoss@unr.edu

Steven Boyce
Portland State University
sboyce@pdx.edu

Beth L. MacDonald
Utah State University
beth.macdonald@usu.edu

Claudia Bertolone-Smith
California State University-Chico
cmbertolone-smith@csuchico.edu

This brief report describes the initial results of The Fractions as Measures for Prospective Elementary School Teachers (PTs) Study. Research shows that PTs' conceptions of fractions on a part-whole understanding may be problematic when teaching children about improper fractions. We created an instructional lesson sequence for PTs focused on using unitizing, iterating, and partitioning to think of fractions as measures in multiple situations. Initial results from this ongoing study indicate that PTs' manipulations of unpartitioned rod manipulatives supported both their construction of more powerful fractions schemes and their ability to verbally justify their reasoning.

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The Standards for Preparing Teachers of Mathematics state that beginning elementary school teachers should have a strong foundation in fractions, including the idea that "fractions have multiple interpretations, including part-whole relationships, measures, quotients, ratios, and operators" (AMTE, 2017). Although the conception of a fraction as a *measure* (Kieren, 1980; Lamon, 2007) is emphasized early in the Common Core State Standards for Mathematics (NGA/CCSSO, 2010), many students and prospective teachers (PTs) remain focused on *part-whole* meanings of fractions (DeWolf & Vosniadou, 2015; Newton, 2008; Norton & Wilkins, 2010; Olanoff, Lo, & Tobias, 2014). In this paper, we present an instructional sequence in an online mathematics methods course for supporting PTs' construction of conceptions of fractions as measures. Our aim of this study is to describe how PTs reason with fractions in this online environment, with the goal of informing design modifications to support measurement conceptions.

Theoretical Framework

We adopt a constructivist epistemology in thinking about PTs' fractions meanings as the product of their organizing mental structures (*schemes*) to fit their experiences (von Glasersfeld, 1995). The construct of *scheme* refers to the way researchers model how individuals operate mentally in service of a goal. A scheme consists of three parts – recognition of a situation, operations (mental actions), and an expected outcome. Individuals' schemes become established as they become refined and generalized through their use, via processes of assimilation and accommodation (Piaget, 1970). When a scheme is interiorized, the situation, operations, and anticipated result are experienced altogether as a unified and connected structure (a concept) that can itself be operated upon (Hackenberg, 2010; Piaget, 1970).

We focus on four specific schemes pertaining to fractions – the part-whole scheme (PWS), the measurement scheme for unit fraction (MSUF), the measurement scheme for proper fractions (MSPF), and the generalized measurement scheme for fractions (GMSF) (Steffe & Olive, 2010; Wilkins & Norton, 2018). The PWS involves *partitioning* a whole into discrete pieces that can be disembedded (removed from the whole without modifying the whole) and double-counted to form a numerosity of part(s) within a numerosity of a whole. The distinction between the PWS and the

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measurement fractions schemes is the individual's ability to iterate fractional units (which is absent from PWS). The MSUF builds upon the PWS as the individual conceives of the *size* of a disembedded unit fraction and its relation to the size of the unpartitioned whole (i.e., that *equipartitioning* an amount of 1 into *n* parts and *iterating* that amount *n* times results in the size of 1). The MSPF extends this notion to the size of a composite (but proper) fraction. An individual with a GMSF understands the size of an (im)proper fraction (*m/n*) as the result of coordinating mental operations to include partitioning the size of '1', disembedding a unit fractional size (1/*n*), and iterating the disembedded fractional unit *m* times.

In order for an individual's fraction scheme to become interiorized as a fraction concept, his or her fraction scheme must be *reversible*. For instance, an individual with a reversible GMSF could reverse his or her ways of operating to determine the size of '1' from a given improper fraction size. Reversing the MSPF involves forming the size of '1' from a given (composite) proper fraction size, and reversing the MSUF involves forming the size of '1' from a given unit fraction size. Reversing the PWS involves forming the numerosity of the whole from a given proper fraction (e.g., reasoning that if three parts represents the fraction 3/7, then the whole must be 7 parts). Wilkins and Norton (2018) explain PWS requires partitioning and disembedding, whereas MSUF also requires iterating. By engaging in both partitioning and iterating, individuals are constructing actions with inverse relationships between each, which also promote reversibility (e.g., partitioning undoes iterating and vice versa when these actions are composed).

Methods

Participants, Context, and Instructional Sequence

We began with a pilot of six PTs enrolled in a face-to-face mathematics methods course to investigate PTs' understandings of linear fraction tasks (Boyce & Moss, 2017). We provided the PTs with unpartitioned rods to help them carry out iterating and partitioning operations for different tasks with fractions. PTs video recorded themselves discussing how to solve particular tasks and showed how they used the rods to help them make sense of each task.

The positive results from the pilot study encouraged us to create an instructional lesson sequence for unitizing, iterating, and partitioning in the linear representation of fractions and expand data collection to an online, asynchronous, undergraduate mathematics methods course for elementary school teachers (*n*=80). This methods course is delivered 100% online for 15 weeks. PTs are typically enrolled in their junior year and one year from a student teaching experience. They complete readings, watch lectures and supplemental videos of whole class instruction, participate in a group discussion, and take a quiz each week. The course content is teaching and learning rational numbers and proportional reasoning.

We designed and implemented the instructional lesson sequence for PTs. The lessons begin with reading the first two chapters of *Fractions into Practice*: *Grades 3-5* (Chval, Lannin, & Jones, 2013). Chapter 1 provides examples of children's work with analyses of children's understanding of partitioning, fair shares, and the meaning of fractions. Chapter 2 provides an overview of developing children's understanding of the meaning of the unit in a fraction. PTs engage in an activity in which they use various pattern blocks to think about partitioning into equal parts. Then, we introduce fractions as measures and the roles of unitizing, iterating, and partitioning in forming sizes consistent with standard and discretized representations (i.e., linear). PTs also engage in thinking about why it is essential for students to identify the unit and recognize its connection with a fractional part. This is accessible to PTs who have not yet constructed fractions as measures. We then foster reflection on iterating and partitioning by asking PTs to complete a series of activities using rods that increase in sophistication and difficulty in terms of thinking of fractions as measures (Baroody, Baroody, & Coslick, 1998, p. 9-16). These tasks encourage use of partitioning, iterating, and unitizing with fractions in order to arrive at an accurate answer. The tasks from the culminating activity, which, use similar wording and form of items from Norton and Wilkins (2010), are listed below:

- If the red rod is 1/5, what rod represents 1/2?
- If the light-green rod is 1/4, what rod represents 1/2?
- If the purple rod is 2/3, what rod represents 1/2?
- If the purple rod is 2/3, what rod represents 7/6?
- If the light-green rod is 1/2, what rod represents 9/6?

We conjectured that PTs might describe reasoning with a MSUF scheme to solve tasks 1 and 2, a MSPF scheme to solve task 3, and a GMSF scheme to solve task 5. We randomly assigned PTs in discussion forums to one of the five aforementioned tasks and asked them to make a video showing their solution(s). PTs posted the video to a discussion forum and discussed the different ways of making sense of their group's assigned task.

Data Collection and Analysis

The design research approach (Cobb, Confrey, diSessa, Lehrere, & Schauble, 2003; Collins, Joseph, & Bielzaczyc, 2004; Kelly, 2003) was used to investigate how PTs reasoned in an online setting and employed to study and understand the means of supporting and organizing student learning of fraction tasks presented in the online course. The framework of design research allowed us to engineer the learning environment, systematically study what takes place, and make adjustments to the curriculum (Cobb et al. 2003; Collins et al., 2004; Kelly, 2003).

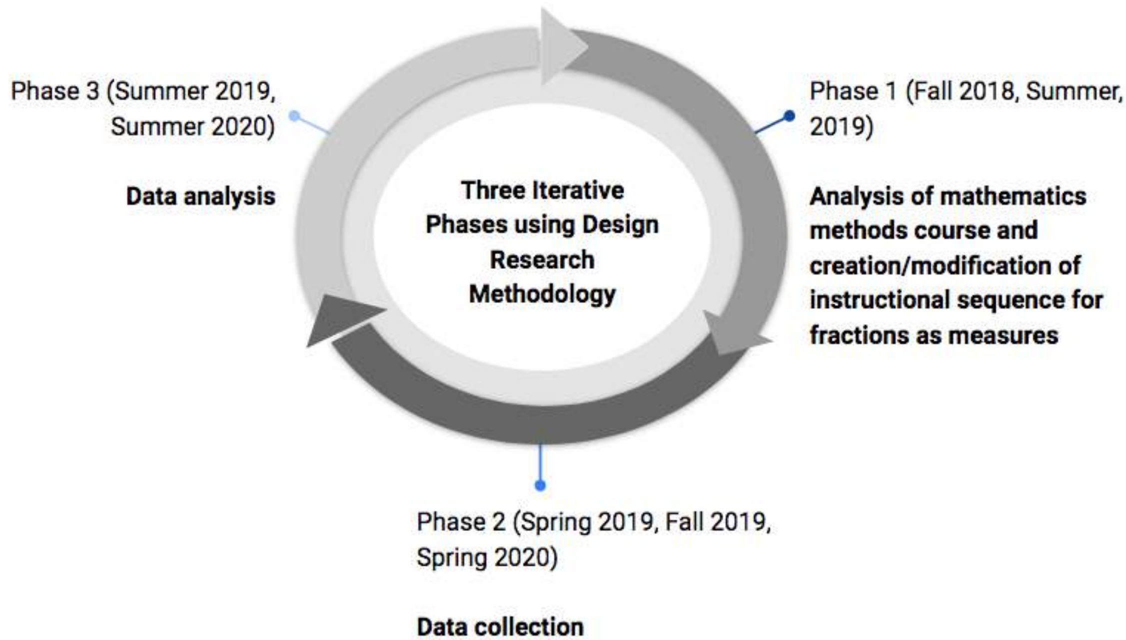

Figure 1: Iterative Phases of Data Collection and Analysis

The data consisted of videos of PTs verbally reflecting on how they solved fraction tasks using rods and PTs' online discussions contrasting their reasoning and their peers' reasoning on these tasks. First, a graduate assistant blinded and renamed the video files and blinded the online discussions so that the five raters could not identify PTs' names. Two researchers identified repeating themes in PTs' videos, while acknowledging unique instances (Corbin & Strauss, 2014). Themes were organized according to "the unit and the whole", "reasoning with arithmetic operations, "alternative methods", and "orientation of the video". The two researchers created a list of twenty questions based on the themes and six researchers quantitatively coded (1: evidenced, 0: not evidenced) in pairs. Following the coding, pairs compared codes to determine inter-rater reliability. Using what we learned from the data analysis, we refined the pedagogy of the online course as well as the content. Figure 1 outlines the phases of our data collection and analysis.

Results

The videos showed that the unpartitioned rods helped PTs verbalize their constructed unit and show how their partitioning and iterating relates to particular fraction reasoning. As PTs began working with concrete manipulative rods, we found that they were able to develop physical actions that were both reversible and composible (e.g. iterating and partitioning). By acting on length models in this way, PTs had to iterate and partition. Before this, many PTs commented that they had used linear representations, but usually in the form of a number line. Thus, conceptualizing fractions with bars using rods was new to them.

Conclusion and Significance to the Field

Results from the study showed that tasks similar to those in the sequence tested provide opportunities for PTs to progress from PWS toward measurement fraction scheme development. By anticipating their own strategies and solutions with (im)proper fractions, PTs will better be able to consider their own students' reasoning strategies more systematically and critically determine (in)effective instructional moves when teaching fractions. By providing PTs means to visualize the unit and engage in reversible actions, and verbally reflect on their thinking (with their video explanations) we posit they might interiorize measurement schemes and be able to anticipate solutions more fluently. Findings from this study could inform (1) (in)effective instructional sequences and progressions that PTs experience when constructing fractions as measures, and (2) affordances and constraints of online asynchronous learning environments for the development of these instructional sequences.

We are currently in Phase 2 of Data Collection and are including new data in the form of drawings, paired with the videos in online and face-to-face mathematics methods courses. We are refining the list of questions based on the themes used to analyze the videos so that other instructors that collect similar data can use the protocol to determine fraction schemes.

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