USING TASK DESIGN METHODOLOGY TO UNPACK TEACHERS' (MIS)CONCEPTIONS ABOUT PROCEDURAL AND CONCEPTUAL KNOWLEDGE

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Star (2005) argues that "current characterizations of the terms procedural knowledge and conceptual knowledge are limiting and are in fact impediments to careful investigation of these constructs" (p. 405). Addressing this argument, we examined secondary mathematics teachers' understanding of procedural and conceptual knowledge through the design of mathematical tasks. We asked 55 secondary mathematics teachers to design a procedural and a conceptual task on a given topic and explain why they think that the task they designed is a procedural and/ or conceptual task. The study results showed that 78% of teachers were able to design and correctly explain procedural tasks. However, only 5.5% of teachers were able to correctly design conceptual tasks. Teachers' narratives were examined to categorize emerging characteristics of procedural and conceptual tasks as well as to address teachers' (mis)conceptions about procedural and conceptual knowledge.

Keywords: procedural knowledge, conceptual knowledge, task design, secondary mathematics teachers.

Objective

Mathematics education reform calls for building students' and teachers' mathematical proficiency that, among other strands, include conceptual understanding and procedural fluency (Kilpatrick, Swafford, & Findell, 2001). Star (2005) suggests that the widespread use of the terms conceptual and procedural in learning and teaching mathematics can be attributed to Hiebert (1986) who defined procedural knowledge as knowledge of procedures (e.g., syntax, steps, conventions, rules) and conceptual knowledge as knowledge of relationships (e.g., connected web of knowledge, a network of linked information). However, there are different interpretations of the conceptual/procedural framework (Star & Stylianides, 2013). Therefore, teachers may agree that reform-oriented mathematics teaching and learning should focus on conceptual knowledge, but it could be difficult to implement "if teachers do not have a common understanding on what conceptual knowledge is" (Star & Stylianides, 2013, p. 5). Considering this challenge, the purpose of this study is to utilize task design methodology as a way to explore secondary mathematics teachers' understanding of procedural and conceptual knowledge. This study addresses the following research question: how do secondary mathematics teachers' operationalize the distinction between procedural and conceptual knowledge through task design methodology?

Perspectives

Procedural vs. Conceptual Knowledge

There is a vast existing literature on the differences between procedural and conceptual knowledge. Star (2005) argues that procedural and conceptual knowledge can have a superficial and/or a deep quality. Deep procedural knowledge is "knowledge of procedures that is associated with comprehension, flexibility, and critical judgment and that is distinct from (but possibly related to) knowledge of concepts" (Star, 2005, p. 408) while deep conceptual knowledge is about knowledge of concepts with rich connections.

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In this study, we examine different ways secondary mathematics teachers express their procedural and conceptual knowledge by evaluating mathematical tasks that they designed. We view the superficial and deep quality of the two types of knowledge as an intersection between procedural and conceptual knowledge, as shown in figure 1. The arrows at the intersection show how the procedural knowledge can deepen into conceptual knowledge and how conceptual knowledge can be surfaced into procedural knowledge.

Task Design

We employed task design as a methodology to unpack teachers' understanding of procedural and conceptual tasks. Research on task design has been common to study teachers' content knowledge (Gellert et al., 2012). Several studies have focused on pre-service teachers' designing tasks as part of their training (Chinnappan & Forrester, 2014; Hannigan et al., 2013; Rayner et al., 2009). Some studies have focused on designing tasks aligned with technology (Gueudet et al., 2016; Hansen et al., 2016; Misfeldt & Zacho, 2016). Additionally, researchers have created tasks to be used by teachers (Jung & Brady, 2016; Tempier, 2016; Wake et al., 2016) while other researchers discussed the design of tasks with teachers (Coles & Brown, 2016; Johnson et al., 2016; Thanheiser et al., 2016). However, when examining mathematical tasks with in-service teachers, most studies have examined how in-service teachers choose mathematical tasks (Cartier et al., 2013; Roth McDuffie & Mather, 2006). The field lacks studies that use task design as a methodology to address teachers' misconceptions. By asking teachers to design their own tasks we can analyze further their reasoning (Cartier et al., 2013).

Methods of Data Collection

Context

This study was part of a larger project that took place during four years from 2013 to 2016. The larger project was a series of professional development workshops focused on mathematics content. This study took place at a university located on the U.S.-Mexico border. The vast majority of people in this area identify themselves as Hispanics (80%). Many of them are recent immigrants from Mexico. The population of the main school districts reflects the demographics of the city. The workshop was aimed to support the training and retention of secondary school mathematics teachers.

Participants

Workshop participants (N=55) were selected from local secondary schools. Teachers that attended the professional development workshop were from five different school districts across the region. Most of the teachers were female (62%). Also, the majority of the teachers reported their race/ethnicity as Hispanics (81%), 17% reported their race as White, Non-Hispanic, and 2% as African American. Years of teaching experience varied from half a year to 15 years.

Data Sources

All 55 teachers that participated in the study answered a survey that required them to design a procedural and conceptual task and explain their reasoning. The purpose of this survey was to examine the teachers' understanding of procedural and conceptual knowledge. Two topics were used for the survey: area and proportion. In addition, we conducted semi-structured interviews where teachers were asked to talk about their understanding of procedural and conceptual tasks.

Data Analysis

Once the survey data was graded, both researchers analyzed the tasks that the teachers designed and looked for patterns. We were interested in examining closely the types of tasks the teachers designed. The tasks were graded on whether they were surface or deep procedural or surface or deep conceptual. We also looked for patterns on their explanations. The interviews were coded to look for instances in which the teachers talked about the design of mathematical tasks. Emergent codes were extracted using linguistic analysis and meaning coding techniques (Kvale & Brinkmann, 2009). After the authors coded the data separately, the two researchers held meetings to reach a consensus on the codes to separate them into final categories.

Results

Teachers created a wide array of tasks as procedural and conceptual tasks. Table 1 shows the percentage of the types of tasks teachers designed. The table clearly shows that teachers were able to correctly design procedural tasks (78%) while the majority designed a procedural task when they intended to design a conceptual task (80%). There is also a percentage of teachers that created tasks that were ill-designed or provided no answer (i.e., 22% for procedural tasks and 14.5% for conceptual tasks). When the explanations were analyzed along with the tasks that teachers designed, we found different patterns. Tables 2 and 3 show the different codes that were created based on teachers' explanations and the types of tasks. The majority of teachers argue that their task is procedural because it includes a procedure (35%) or because it requires to substitute or "plug-in" values in a formula (29%). When they were designing the conceptual task they argued that their task is conceptual because: it is about finding a relationship (26%), it is a multi-step problem (26%), it is a word problem (23%) or it has a real-world connection (21%).

Besides designing a task teachers had to provide a solution as well as an explanation of why they think the task is either procedural or conceptual. For example, a teacher designed the following task: "solve the following, $\frac{x}{3} = \frac{7}{20}$" as a procedural task. This teacher wrote the following explanation: "Must find x using cross multiplication, then division, very procedural, no connection." For this teacher, this problem is procedural because is about just solving for x. Using Star's (2005) classification, we rated it as a superficially procedural task, The following task was intended as a conceptual task:

Laura types 168 words in 25 minutes, if she continues typing at this rate, how much time will she spend typing a 1500 word paper?

The explanation for this task written by the teacher was: "Because students need to apply what they learned on proportions by solving real-world problems in order to make connections." Based on this teacher's explanation, there is some understanding about conceptual knowledge by using words and phrases like "real-world problem" and "connections." However, upon further examination of this task, we can see that this task requires just procedural knowledge since, after setting up the equation, the solution would look very similar to the previous one. The main difference is that this is a word problem, which would require a student to read the problem, determine if this is a proportional situation, and set up the equation. Therefore, we rated it as a deep procedural task using Star's (2005) classification.

Another teacher designed the following task: "What is the maximum area of a rectangle if the perimeter is 20?" with an explanation that said, "it requires to use prior knowledge of area and perimeter". The use of "prior knowledge" in the explanation might imply that the teacher was thinking about how the student would have to make connections between fixed perimeter and changing area. This task was one of the few that was rated as a deep conceptual task following Star's (2005) classification. During interviews, teachers expressed the desire to design more conceptual tasks but said they need help. For instance, a teacher said about conceptual tasks, "to get them (students) to apply it to the real world and forces them to kind of make the connections, so it's something I think I am improving on, I don't think I am quite at the area but I am improving on it…"

Discussion and Conclusions

Teachers have an understanding of procedural knowledge related to the steps that require solving a mathematical task. While some teachers used language related to conceptual knowledge in their explanations, they face challenges in designing conceptual tasks. For the majority, the actual tasks that they designed illustrate some misconceptions about procedural and conceptual knowledge. This study adds to the growing literature about procedural and conceptual knowledge (Hannigan et al., 2013; Rayner et al., 2009; Rittle-Johnson et al., 2015; Star & Stylianides, 2013) by utilizing task design as a methodology. Based on teachers' (mis)conceptions of procedural and conceptual tasks, more studies need to be conducted to aid teachers not only in selecting tasks but in designing them as well.

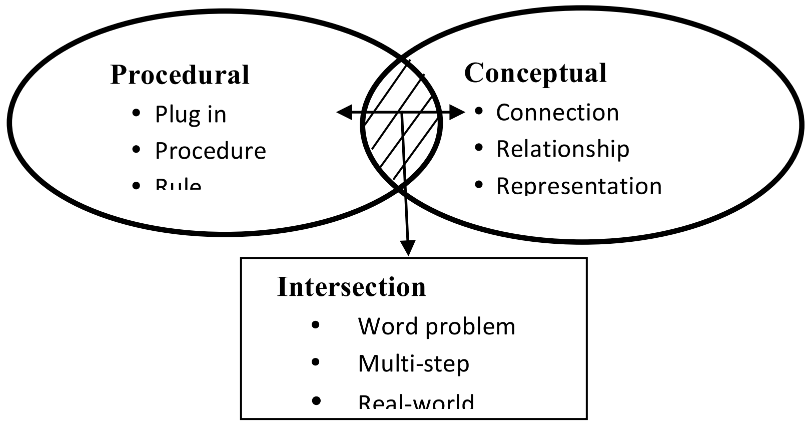

Figure 1: Relationship between Conceptual and Procedural Tasks

Table 1: Teacher designed tasks rated by experts as procedural and/or conceptual

	Procedural	**Conceptual**
Rated as Procedural	43 (78%)	44 (80%)
Rated as Conceptual	0 (0.0%)	3 (5.5%)
Ill-designed/no answer	12 (22%)	8 (14.5%)
Total	55 (100%)	55 (100%)

Table 2: Number and Percentage of Teacher Explanations for the Procedural Task

Procedure	**Word-Problem**	**Substitute**	**Multi-step**	**No real world connections**	**No explanation**	**Total codes**
18 (35%)	8 (16%)	15 (29%)	4 (8%)	6 (12%)	0 (0%)	51 (100%)

Table 3: Number and Percentage of Teacher Explanations for the Conceptual Task

Procedure	**Word-Problem**	**Substitute**	**Multi-step**	**Real world connection**	**No explanation**	**Total codes**
12 (26%)	11 (23%)	1 (2%)	12 (26%)	10 (21%)	1 (2%)	47 (100%)

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