PROSPECTIVE TEACHERS' INTERPRETATIONS OF MATHEMATICAL REASONING

Hyejin Park
Marquette University
hyejin.park@marquette.edu

Marta T. Magiera
Marquette University
marta.magiera@marquette.edu

Calls for teaching school mathematics with a focus on mathematical reasoning (MR) are included in curricular documents across the world, but little is known how prospective teachers (PSTs) understand MR. In this paper, we report on a study in which we engaged 24 PSTs preparing to teach grades 1-8 in analyzing a series of student-generated arguments for evidence of student reasoning with a focus on student-provided justifications. We examined PSTs' interpretations of MR prior to and after instruction. Our results showed that PSTs interpreted MR broadly in terms of student thinking, validating thinking, problem-solving, connecting ideas, or sense-making. Some PSTs also interpreted MR as evidence of student understanding or described MR in terms of strategies teachers use to support students' reasoning skills. We discuss changes in PSTs' interpretations of MR after instruction.

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Framing of the Study

Developing students' mathematical reasoning (MR) skills is the desired goal of school mathematics education (Australian Curriculum, Assessment and Reporting Authority, 2015; Department for Education, 2014; National Council of Teachers of Mathematics [NCTM], 2000, 2009; National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). However, little is known about how teachers, including prospective teachers' (PSTs'), interpret MR. To date, only a handful of studies documented how practicing and PSTs make sense of and understand MR (Clarke et al., 2012; Herbert et al., 2015). Herbert et al. (2015) shared that Australian and Canadian elementary practicing teachers have broad and ambiguous perceptions of MR. As such, they interpreted MR as thinking, communicating thinking, problem-solving, validating thinking, forming conjectures, using logical arguments to validate conjectures or connecting aspects of mathematics.

In this paper, we describe instructional intervention designed to heighten elementary PSTs' attention to students' reasoning in the context of generating mathematical arguments. Our study draws on the variation theory of learning (Lo, 2012), which highlights the importance of providing learners with multiple experiences with a given phenomenon, to generate a wide range of opportunities that help learners attend to and make sense of different features of that phenomenon. In our work with PSTs, we draw on the variation theory to purposefully bring PSTs' attention to elementary students' MR, particularly different ways in which students might reason to justify while generating mathematical arguments. Our goal was to answer the following research questions: (1) How do PSTs interpret MR in the context of elementary school mathematics classrooms? And, (2) How does engaging PSTs in analyzing elementary students' arguments for evidence of MR impacts PSTs' views on MR?

Method

Participants and Study Context

Participants were 24 PSTs enrolled in a semester-long mathematics content course for elementary and middle grades education majors, Algebra and Geometry for Teachers. The course was designed to support PSTs' conceptual understanding of mathematical ideas essential to elementary and middle school mathematics curriculum. Instructional emphasis was placed on understanding, interpreting, and assessing students' MR about fundamental mathematics concepts in the K-8 school mathematics.

Drawing on descriptions of elementary students' reasoning provided by the NRICH team at the University of Cambridge (see https://nrich.maths.org/11336) and the variation theory of learning, we designed the Student Reasoning Assessment Tool (SRAT) (see Table 1) to bring PSTs' attention to different justifying actions in the context of student-generated arguments and to help PSTs develop reasoning assessment skills. Along with the SRAT, we also created a set of class activities that we used to engage PSTs in analyzing MR evident in elementary students' written arguments.

Table 1: Student Reasoning Assessment Tool (SRAT)

Levels	Descriptions of elementary students' reasoning levels
L0	Student tells what he or she did
L1	Student attempts to provide some reasoning (not necessarily relevant, complete, or valid) for what he or she did
L2	Student provides a chain of reasoning, which is incomplete, insufficient, or invalid, to support the assertion
L3	Student provides a chain of acceptable valid reasoning in support of the assertion; the argument is at best partial
L4	Student provides an exhaustive acceptable chain of valid reasoning in support of the assertion; the argument can be accepted as proof

Prior to class intervention, we asked PSTs to share in writing their own interpretations of MR. They were also given a set of student-generated arguments and asked to analyze these arguments for evidence of student reasoning with attention to student-provided justifications. During the intervention, using the SRAT, individually and in small groups, PSTs examined a wide collection of sample arguments for evidence of student reasoning. They were also asked to anticipate how elementary students could reason and communicate their mathematical reasoning in different problem contexts. After the intervention, we asked PSTs to revisit their initial descriptions of MR.

Data and Data Analysis

We analyzed PSTs' written responses to two journals, which each PST completed at the beginning and end of the semester and in which they reflected on the meaning of MR. The journal prompts were intentionally open-ended to avoid leading PSTs in any specific direction that could suggest interpretations of MR. The prompts were as follows:

- Journal 1: Think about yourself as a mathematics teacher. When you hear the term mathematical reasoning, what does it mean to you? In the best possible way, describe your understanding of this term. Explain how mathematical reasoning might look.
- Journal 2: Building on your learning in this class, define mathematical reasoning. Did your understanding of mathematical reasoning change when comparing to how you interpreted it at the beginning of the semester? If yes, explain why. If no, explain why not.

We first coded the data with analytic codes derived from the existing literature on teachers' perceptions of MR (e.g., Davis & Osler, 2013; Herbert et al., 2015). We then conducted the inductive analysis to identify any additional themes within our PSTs' responses. We continued comparing and contrasting the identified themes until we established final definitions of codes, which then were applied to our data. Finally, we tabulated code frequencies to identify any patterns in our PSTs' interpretations of MR.

Results

PSTs' Interpretations of MR

Our PSTs interpreted MR in multiple ways, and that overall, they used two lenses while discussing the term MR. Most frequently, PSTs interpreted MR from the perspective of a learner (see Table 2). Some PSTs also viewed MR from the perspective of a teacher. PSTs with the learner perspective saw MR as the process that describes how students think, validate (justify), make sense, solve problems, or connect mathematical ideas. PSTs with the teacher's perspective viewed MR as data (products) that give teachers evidence of students' understanding of mathematical concepts, or as strategies with which teachers engage students in MR in the mathematics classroom.

Table 2: PSTs' Views of MR

View	Interpretation of MR	The number of PSTs* with the view (n, %)	
		Semester beginning	Semester end
Student-Centered	Thinking	6 (25%)	10 (42%)
	Validating thinking	18 (75%)	22 (92%)
	Sense-making	13 (54%)	17 (71%)
	Problem-solving	8 (33%)	9 (38%)
	Connecting mathematical ideas	9 (38%)	14 (58%)
Teacher-Centered	Evidence of students' understanding	2 (8%)	7 (29%)
	Teacher support for students' reasoning	3 (13%)	3 (13%)

Note. The total number of participants, n = 24. The categories are not mutually exclusive. Most PSTs shared multiple views.

Student-centered Views of MR

With a focus on an individual student, PSTs most frequently viewed MR as validating thinking. They emphasized justifying actions as representative of MR. PSTs also discussed modes of representations (e.g., verbal, written, or pictorial forms) that students might use to validate or explain their reasoning. They focused on the role that reasoning plays in supporting the growth of one's mathematical understanding by describing that while students reason about mathematics, they learn and develop a deeper understanding of mathematical concepts or problem-solving strategies. PSTs also viewed MR as specific aspects of the problem-solving process, the entire process of problem-solving, or decision-making in problem-solving situations. Some PSTs described MR as one's thinking about mathematics, mathematical problems, or specific problem-solving strategies.

Teacher-centered Views of MR

PSTs interpreted MR as evidence of student learning and articulated that teachers use students' reasoning as a resource for making instructional decisions. PSTs also described that by paying attention to students' reasoning, teachers identify the needs of students with diverse mathematical abilities or levels of understanding.

Changes in PSTs' Interpretations after Class Intervention

We observed two changes in PSTs' interpretations of MR while comparing their views from the beginning to the end of the semester. (1) Change in the breadth of interpretations (17 PSTs, 70%). After the class intervention, many of the PSTs augmented their initial interpretations and included additional perspectives on the meaning of MR, which they did not initially consider. On average, after the intervention, most PSTs gained awareness of one to three additional interpretations of MR. (2) Change in the depth of interpretations (14 PSTs, 58%). PSTs' views of MR after the class intervention remained largely consistent with their initial views. However, their interpretations of MR were more nuanced and included more precise descriptions of reasoning actions. While describing justifying prior to the class intervention, for example, many of the PSTs interpreted justifying broadly as explaining why. After the class intervention, PSTs discussed justifying with attention to specific attributes of justifications such as logic, generality, or modes of representations that a student might use to justify a mathematical statement.

Conclusion and Discussion

In our work with PSTs, we positioned them to analyze students' MR as future mathematics teachers (see presented earlier journal prompts). Our data revealed two perspectives that PSTs used as their lenses while describing MR: student-centered and teacher-centered. With a focus on each of these perspectives, PSTs interpreted MR in a broad sense. Within their student-centered interpretations, PSTs described MR as thinking, validating thinking, sense-making, problem-solving, or connecting mathematical ideas. Within their teacher-centered interpretations, PSTs interpreted MR as evidence of student learning that helps teachers make instructional decisions or as a pedagogical practice that teachers use to engage students in reasoning and encourage their mathematical thinking. PSTs' broad interpretations of MR might not be surprising since reasoning, problem-solving, sense-making, mathematical thinking are all intertwined and often viewed as interconnected practices that support one another (NCTM, 2009; Kilpatrick et al., 2001).

Our results also revealed that classroom activities that exposed PSTs to a large sample of students' work with a focus on student reasoning about justifications increased PSTs' awareness of specific justifying actions. While sharing their views of MR, PSTs have begun to provide more nuanced and precise descriptions of reasoning actions related to justifying. Supporting PSTs in building a comprehensive vision of MR should include efforts of helping them make a shift from a broad understanding of MR as thinking, sense-making, problem-solving, or connecting mathematical ideas to seeing these aspects of reasoning in terms of more specific and tangible reasoning actions. Loong and colleagues (2013) argued that teachers who do not have a strong understanding of specific reasoning actions might likely be ineffective in promoting MR in their classrooms.

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