THE EVOLUTION OF TEACHERS' CONCEPTIONS OF TEACHING MATHEMATICAL MODELING THROUGH PARTICIPATION IN LESSON STUDY

Jenifer Hummer
West Chester University
jhummer@wcupa.edu

Mathematical modeling is an important mathematical practice, yet it is a relatively new idea in school mathematics. Limited resources such as lack of teacher preparation have contributed to challenges of teaching modeling. Thus, professional development, such as lesson study, a continuous improvement approach to teaching, might support teachers in implementing modeling. For this study, three secondary teachers with varying levels of experience participated in two cycles of lesson study on mathematical modeling. The teachers were interviewed about their conceptions of teaching mathematical modeling before and after the lesson study. Analysis of the interview transcripts revealed that the teachers' conceptions of teaching modeling evolved in ways that indicated the teachers learned pedagogical strategies, realized further benefits of teaching modeling, and refined their instructional focus for teaching mathematical modeling.

Keywords: Modeling, Teacher Education – Inservice / Professional Development

Policy documents have called for the incorporation of mathematical modeling into the curriculum (e.g., NCTM 2000, 1989; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Yet, research has indicated that teachers may have challenges with teaching this important mathematical process that is becoming increasingly more prominent in science, technology, engineering, and mathematics fields (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016). The challenges of teaching modeling include limited experience, teachers' dispositions and lack of resources (e.g., Meyer, 2015; Newton, Madea, Alexander, & Senk, 2014). These challenges are unfortunate because mathematical modeling can provide students with opportunities to engage in ill-structured problems about authentic real-world situations that are unlike typical textbook tasks. Thus, researchers have recommended that teachers receive professional development (PD), such as lesson study, on implementing mathematical modeling (Turner et al., 2014).

In previous studies lesson study has supported teachers to improve their teaching (e.g., Lewis, 2016; Murata, Bofferding, Pothen, Taylor, & Wischnia, 2012). Lesson study is an iterative process that includes curriculum study, lesson planning, teaching and observing, and reflecting/debriefing. Lesson study could support the teaching of mathematical modeling because lesson study employs a variety of tools and resources that influence improvement of teaching (see e.g., Lewis, C., 2016; Takahashi & McDougal, 2016.) Hence, this study sought to understand: What are teachers' conceptions of teaching mathematical modeling after participating in lesson study on mathematical modeling?

What is Mathematical Modeling?

There are many descriptions of mathematical modeling, but there is no agreed-upon definition. Hence, for the purpose of this study, a working definition of *mathematical modeling* is inspired by Cirillo, Pelesko, Felton-Koestler, and Rubel's (2016) description of the features of mathematical modeling: Mathematical modeling is an iterative process that authentically connects to the real world. It is used to explain phenomena in the real world and/or make predictions about the future behavior of a system in the real world. Mathematical modeling requires creativity and making choices, assumptions, and decisions, and can have multiple approaches and solutions.

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Challenges of Teaching Mathematical Modeling

A review of literature revealed multiple challenges for teaching mathematical modeling. For one, preservice and inservice secondary mathematics teachers reported a lack of self-efficacy with respect to pedagogical strategies for teaching mathematical modeling, specifically because of the ill-structured nature of modeling activities (e.g., Kuntze, Siller, & Vogl, 2013; Chan, 2013). Other researchers observed that teachers have struggled with anticipating multiple student responses for mathematical modeling activities. This pedagogical challenge led to other obstacles with respect to classroom management, the handling of multiple student approaches, and the facilitation of whole-class discussions (e.g., Pereira de Oliveria & Barbosa, 2013; Borromeo Ferri & Blum, 2013). These challenges indicate that teachers need further preparation to teach mathematical modeling.

A need for teacher preparation is particularly true when considering the US context. Evidence of insufficient teacher preparation was revealed through a survey of education programs (n = 72). Of the respondents, only 15% of the programs required a mathematical modeling course for preservice secondary teachers (Newton, Madea, Alexander, & Senk, 2014). This finding could indicate that novice secondary teachers may not implement modeling as recommended (see e.g., CCSSM). Gould (2013) found, in a national survey of inservice teachers (n = 274) from 35 states that teachers held misconceptions about mathematical modeling. While Gould's (2013) results indicate that teachers may lack content knowledge for teaching modeling, teachers may also hold beliefs that impact their willingness to teach mathematical modeling. As an example, Anhalt, Cortez, and Bennett (2018) found that after completing a modeling activity, preservice teachers acknowledged that implementing modeling could provide many opportunities for students to engage in rigorous mathematics, rich discussions, and develop multiple solution approaches. At the same time, those teachers were concerned that mathematical modeling activities would be too complex for most secondary students. These types of beliefs may hinder teachers from implementing modeling activities. Moreover, inservice teachers have reported limited resources, such as time and access to quality mathematical modeling curriculum materials prevented them from implementing mathematical modeling regularly, if at all (Gould, 2013; Huson, 2016). To address these challenges researchers have recommended lesson study as a means of professional development to support the teaching of mathematical modeling.

Using Lesson Study to Address Challenges of Teaching Mathematical Modeling

Through the process of lesson study, teachers have opportunities to learn from each other while collaboratively: planning, observing teaching, and debriefing to improve lesson plans (Lewis et al., 2009). Previous studies have found that lesson study provides opportunities for teachers to improve content knowledge, learn pedagogical strategies, and focus on student thinking (e.g., Murata et al., 2012; Lewis et al., 2009). This focus on student thinking can influence teacher learning (e.g., Suh & Seshaiyer, 2014). For instance, teachers observed by Inoue (2011) drew on anticipated student responses to facilitate discussions and support students' engagement with mathematical reasoning. Additionally, Cajkler et al. (2015) observed how teachers' beliefs changed after reflecting on their observations of student thinking for students who typically had minimal classroom participation. These aforementioned aspects of lesson study are likely to support teachers in the implementation of open-ended modeling activities.

Methods

Setting and Participants

Three secondary teachers who were teaching in a diverse vocational high school in the mid-Atlantic region of the United States were recruited based on their interest in improving their teaching of mathematical modeling. Loren, a second-year teacher, earned her bachelor's degree in secondary

mathematics education, and as part of her degree program, completed one course on mathematical modeling for secondary teachers. Anne, who had previously been an engineer, had six years of teaching experience. Anne held a bachelor's degree in electrical engineering and a master's degree in curriculum and instruction. Karen had 21 years of experience teaching and had earned a bachelor's degree in computer information systems and a master's degree in education. Karen had participated in professional development on mathematical modeling, and she had some experience teaching modeling.

Lesson Study on Mathematical Modeling. The teachers participated in two lesson study cycles containing the following activities: curriculum study, lesson planning, teaching and observing, and debriefing. During the curriculum study, the researcher introduced mathematical modeling, and the teachers explored curriculum materials related to modeling. For instance, the teachers explored tasks such as those provided in the *Mathematical Modeling Handbook* (Gould, Murray, & Sanfratello, 2012). Next, the researcher facilitated planning meetings and guided teachers in completing an annotated lesson plan, similar to a common format used in Japanese lesson study (e.g., Gorman et al., 2010; Lewis & Hurd, 2011). The lesson plan template contained cells for learning goals, anticipated student responses, planned instructor actions, and rationale for tasks. To complete the lesson study cycles, each teacher enacted the lesson while the lesson study team observed. Then, the team met to debrief the lesson enactments after the first enactment (i.e., Loren's) and the third enactment (i.e., Karen's). During the debrief sessions, the researcher executed a debrief protocol to support revision of the lesson plan based on evidence of student thinking collected by the teachers while observing each other's enactments.

Interview Protocol

Interviews were conducted before and after the lesson study, regarding the teachers' conceptions of teaching mathematical modeling. The questions provided teachers with opportunities to share their conceptions of teaching mathematical modeling. For example, one of the questions was: "Are you currently teaching mathematical modeling, or have you ever taught mathematical modeling? Describe your teaching approach to mathematical modeling (e.g., frequency, resources for tasks, aspects of the modeling cycle addressed)." Then follow-up questions were asked about teaching approaches to modeling, including the benefits and challenges of teaching mathematical modeling. Prior to the lesson study, if the teacher was not currently teaching modeling, then the teacher was asked why modeling was not being taught, and to describe any hypothetical benefits and challenges of teaching modeling.

Data Analysis

Data for the study consisted of audio-recordings and transcripts of two interviews per teacher for a total of six interviews. Once the audio data were transcribed the transcripts were uploaded to Dedoose (2016), web-based qualitative data analysis software. The transcripts were analyzed using a constant comparative approach (e.g., Strauss, 1987; Hatch, 2002). Initial deductive codes were developed using themes that emerged in the literature with respect to the working definition of modeling as well as benefits and challenges of teaching modeling. Then the coding dictionary was revised further as inductive codes emerged from the data. Themes were organized into analytic memos to inform the findings. The analysis of the data revealed three cases about the teachers' conceptions of teaching mathematical modeling: Loren learned pedagogical strategies for teaching mathematical modeling; Anne realized the benefits of teaching mathematical modeling; and Karen focused on shifting her classroom culture.

Findings

Case 1 – Loren: Learned Pedagogical Strategies for Teaching Mathematical Modeling

Before the Lesson Study.

In the pre-lesson study interview, Loren shared that she had not taught authentic mathematical modeling yet, but she expressed several potential benefits and her concerns for challenges of teaching mathematical modeling. For example, she mentioned that teaching mathematical modeling "makes mathematical modeling more interesting to students" and that students could have opportunities to be creative and to see multiple approaches to a modeling activity. Although Loren acknowledged potential benefits of teaching modeling, she also mentioned that modeling activities could be "complicated" and that students could get "easily frustrated" while engaging in modeling. She seemed to think that the complexity of modeling activities could be an obstacle for students. Even though Loren had not been teaching modeling, she indicated that she attempted to provide opportunities to experience similar benefits through her teaching of word problems.

As mentioned earlier, Loren had limited experience with teaching mathematical modeling. She described her approach to teaching modeling as implementing "word problems." When reflecting on her experience teaching modeling she said:

> The only experience that I've really had, and I don't even know if you would classify it as modeling is just like drawing pictures and setting up scenarios with word problems…I'm thinking of when I student taught geometry, and we would do those drawings with the ladder leaning on the house, and I've taught students ways to show mathematical situations, but I don't know if it's truly modeling.

Here, Loren acknowledged that her use of word problems was not quite modeling even though students had opportunities to apply multiple mathematical representations in their solutions. Loren also conveyed enthusiasm for participating in lesson study and improving her teaching of mathematical modeling so that she could genuinely engage her students in the benefits of teaching mathematical modeling.

After the Lesson Study. After participating in lesson study, Loren's observations of students in her classroom and her two colleagues' classrooms influenced her conceptions of the benefits of teaching mathematical modeling. While she mentioned several benefits in her initial interview, Loren added that when the students engaged in modeling during the lesson study, she observed them engage in mathematics that was "valuable in the real world" and "really relevant." She observed that teaching modeling was "a lot more rigorous for them than just teaching them how to do procedures." Loren also recognized how the modeling tasks provided opportunities for students to collaborate with their classmates. Prior to the lesson study, Loren was concerned that mathematical modeling activities would be too complex. However, after focusing on student thinking during lesson study, she saw how students could persevere and collaborate to produce multiple solution pathways for mathematical modeling activities.

Loren's new experience with teaching mathematical modeling also exposed some worthwhile challenges of teaching modeling. Loren found it challenging to support students with "group roles" while they collaborated on the modeling tasks. She also worried about "tutoring [students] too much." Thus, to avoid too much "telling," Loren said she relied on the collaboratively planned lesson which was annotated with possible student responses and questions to ask students. This type of lesson planning seemed to influence Loren's teaching approach to mathematical modeling as she indicated in the transcript below.

> It [lesson planning] really showed me how valuable that is, and I like so enjoyed like going into [the lesson] knowing, what I wanted to say, what I didn't want to say, and what I thought the students

> would say. I planned timing and things like that, but the lesson plan that we did went in depth which was great. So, I think it'll change the way I teach. Not only teaching modeling but just teaching in general because it showed me the importance of lesson planning.

As a new teacher, the lesson planning activities and focus on student thinking during lesson study influenced her approach to teaching mathematical modeling. While Loren expressed some concerns about the challenges of teaching mathematical modeling, she also seemed to indicate that her new knowledge of pedagogical strategies would continue to support her teaching.

Case 2 – Anne: Realized the Benefits of Teaching Mathematical Modeling Before the Lesson Study.

Similarly, to Loren, Anne mentioned that she had not quite taught mathematical modeling before, but she acknowledged potential benefits and challenges of teaching mathematical modeling. Anne, conveyed that mathematical modeling could advance students' mathematical thinking and that teaching modeling was "teaching them more the approach to solving problems rather than just rote memorization of solving problems, different representations and ways to solve things, but mostly just to get them thinking more analytically rather than plugging things into a formula." While Anne recognized that modeling could provide students opportunities to engage in complex mathematics, she also indicated that students could become frustrated with ill-structured modeling tasks. Anne noted that when she had implemented open-ended mathematical tasks her "biggest challenge" was "letting them [students] struggle." She also mentioned that the students were uneasy with tasks that could have multiple student responses and not being "spoon-fed, step-by-step" instructions for solving the tasks. Anne seemed to think that struggling with mathematics would be beneficial for students, but she also mentioned her frustration with the lack of time for including open-ended tasks within her mandated curriculum when she said "It's just sometimes it's hard with so much to get through. There's definitely room for more."

After the Lesson Study. After the lesson study, Anne still maintained her perceived benefits of teaching mathematical modeling. She indicated that she saw students in her classroom, and her two colleagues' classrooms "get better at problem solving." She also thought those skills would translate to other areas outside of the math classroom. A new conception that Anne mentioned was based on her observations of students. She saw that engaging in mathematical modeling gives students opportunities to "collaborate with other students and things like that rather than just being instructed directly." Anne's evolved conceptions about her students were notable. After the lesson study and implementation of mathematical modeling, she discussed how the intentional focus on student thinking allowed her to observe "skills in students that [she] wouldn't have seen in a traditional way of teaching them." More notably, she had the following to say about one of her students in particular:

> One of my students, like I got more out of him from this activity than I have the whole semester. I was able to see his thought process and things like that, that I had never seen before because I guess really, I'm always looking at a paper and what he's writing, and he's not a big sharer in class. So, it was really hard for me to see. But then when I saw what he was doing I was like, wow, he's really, really working on this. Like really his, the way his mind was working, was very much different than what I had thought.

Anne's discussion about her observations of this particular student indicated that her initial beliefs, regarding students' abilities to engage with mathematical modeling, had evolved to see that students could persevere through modeling activities. As a result, Anne mentioned that she was including more modeling tasks and asking more open-ended questions throughout her lessons. Through her participation in the lesson study, Anne's conceptions evolved so that she had a deeper realization of the benefits of teaching mathematical modeling.

Case 3 – Karen: Focused on Shifting Classroom Culture

Before the Lesson Study. As the most experienced teacher, Karen, was well-aware of many benefits and challenges of teaching mathematical modeling prior to the lesson study. When describing the benefits of modeling, she conveyed her implementation of modeling had evolved. For instance, she mentioned how students struggled with the messiness of modeling tasks at first, but then "eventually they started to deeply think about it and contribute to each other's ideas and bounce ideas off of each other and reference each other's input as a class discussion." Because of her previous experience with teaching modeling, unlike her two colleagues, she had a clear vision for implementing modeling. Although Karen recognized many benefits to teaching modeling, she also noted challenges with teaching modeling. Karen's primary concerns about teaching mathematical modeling were with regard to her available resources. She indicated that when it comes to teaching mathematical modeling, "the primary constraint is the lack of great tasks and the lack of time." Karen shared that she implemented a modeling activity within each of her units, but she was looking forward to the opportunity to collaborate with colleagues to further improve her teaching of modeling. Although Karen's conceptions of teaching modeling did not evolve as drastically as her other two colleagues' conceptions, she still found value in the lesson study and her conceptions of teaching modeling emerged to a different teaching focus.

After the Lesson Study. As hypothesized by the researcher, many of Karen's conceptions about the benefits and challenges of teaching mathematical modeling did not change much after teaching the lesson study. Rather than observing noticeable changes in her conceptions, the researcher noted that Karen's focus after the lesson study had moved to cultural aspects of teaching mathematical modeling. For example, Karen noted that she appreciated the level of student engagement she observed during the lesson enactments. She also spoke about her evolved focus on cultivating a classroom culture that would support the teaching of mathematical modeling. Specifically, she said she was challenged with:

> Creating that environment where they want to do this type of math is an ongoing challenge of mine…just getting [students] to be comfortable with being uncomfortable is what I try and get to tell them. I am becoming more and more aware of the importance of establishing appropriate culture, and it doesn't matter what the task is or how wonderful my task is. If I can't get the students to buy what I'm selling, it's not, it's just not going to have the impact that I wanted to have. I need to have these kids believing that they can model.

It seemed as though Karen was less focused on her earlier challenges with limited resources for teaching modeling, and more focused on her teaching approach. She also noted other benefits of participating in the lesson study.

During the post-lesson study interview, Karen expressed that she found the lesson study to be beneficial for multiple reasons. First of all, even though she was the most experienced teacher, she found that the collaborative nature of lesson study supported her to anticipate student thinking for the implemented lessons. Consequently, she communicated that she would like to continue to collaborate with teachers in her school to improve her teaching of mathematical modeling. More specifically, Karen said she appreciated the " ability to plan with someone else and anticipate [student thinking], regardless if it's modeling or not modeling, but I wish I had that with my modeling tasks. I know my modeling tasks would be improved if I could do that." Even though Karen was a veteran teacher, she indicated an aspiration to collaborate with her colleagues in the future to improve her teaching of mathematical modeling.

Reflections Across the Cases

Before the Lesson Study. Prior to the lesson study, all three of the teachers acknowledged that teaching mathematical modeling could have many benefits for students but also challenges for

students and teachers (see Table 1). Loren, who had engaged with modeling as a student, and Karen, who had been teaching modeling, both recognized that teaching mathematical modeling could appeal to student interest. Anne also indicated that engaging in modeling would provide more rigorous problem-solving opportunities than typical textbook texts usually provide. Yet, at the same time, Loren and Anne were concerned that the complexity of modeling could discourage some students. Notably, Karen did not focus on challenges for students, but she expressed challenges she faced as a teacher with finding authentic modeling tasks and time to plan and implement mathematical modeling. At the beginning of the study, as expected, the teachers were approaching the teaching of mathematical modeling in various ways, as Loren and Anne had not quite implemented open-ended modeling tasks. When considering, the teachers' early conceptions of teaching modeling, their evolved conceptions after the lesson study suggested that the lesson study had a positive impact on their conceptions of teaching mathematical modeling.

Table 1:Teachers' Conceptions of Teaching Mathematical Modeling

		Benefits	Challenges	Teaching Approach
Loren	Before	• Student interest • Creativity • Multiple approaches	• Complexity of modeling	• Word-Problems
	After	• Relevant math • Rigorous math • Student collaboration	• Time • Curriculum	• Focus on Student Thinking • Group Roles
Anne	Before	• Problem-solving	• Complexity of modeling	• Word-problems
	After	• Student collaboration • Access for all students • Focus on student thinking	• Culture Shift	• Increased modeling tasks • Open-ended questions
Karen	Before	• Student interest • Student collaboration	• Time • Curriculum	• Modeling in unit plans • Adapting tasks
	After	• Rigor and relevancy • Student collaboration	• Culture Shift	• Collegial Collaboration • Culture Shift

After the Lesson Study. After the lesson study, the teachers' expressions conveyed evidence that indicated their conceptions of teaching mathematical modeling had evolved. As an example, Loren and Anne's concerns about the complexity of modeling tasks being too challenging for students had shifted. Instead, they expressed how teaching modeling provided opportunities to focus on student thinking and observe how all of their students could engage in rigorous mathematics. Similar to Karen, before the lesson study, Loren now conveyed curriculum and time as being her main challenges to implementing modeling. Likewise, Anne's new perceived challenges were aligned with Karen's challenges of creating a classroom culture for teaching modeling. Additionally, Loren and Anne had other conceptions after the lesson study about the benefits of modeling that were similar to Karen's conceptions before the study. For example, before the lesson study, Karen indicated that modeling could appeal to student interest and promote student collaboration; whereas, Loren and Anne did not mention those benefits until after the lesson study. When considering the cross-case findings, it is apparent that the evolution of the teachers' conceptions about teaching mathematical modeling was supported by participating in the activities of lesson study. More importantly, one might hypothesize that Loren's and Anne's conceptions of teaching mathematical modeling progressed more expediently than if Loren and Anne had attempted to implement mathematical modeling on their own or through conventional PD.

Discussion and Conclusions

The teachers' conceptions about teaching mathematical modeling after participating in lesson study indicated that lesson study can, given the right conditions, support the teaching of mathematical modeling. For one, the teachers indicated that participating in lesson study provided opportunities to explore curricular resources and learn new pedagogical strategies. They also had time to collaborate with colleagues to improve mathematical modeling lessons. This finding is important as teachers in previous studies have indicated that they had limited resources for teaching mathematical modeling such as curriculum and time (Gould, 2013; Huson, 2016). The teachers in this study also focused on how their implementation of mathematical modeling provided multiple benefits for students. The teachers spoke about how the lesson study provided opportunities to engage with student thinking in ways that supported students' engagement in the modeling process. This finding is contrary to previous studies where teachers struggled with pedagogical skills needed for teaching modeling such as facilitating multiple student responses (e.g., Kuntz, Siller & Vogl, 2013; Pereira de Oliveria & Barbosa, 2013; Borromeo Ferri & Blum, 2013). Another notable finding from this study was that the teachers observed how all students were capable of engaging in rigorous mathematics. In contrast, teachers in previous studies expressed concerns about students' abilities to engage in complex mathematical modeling tasks (e.g., Anhalt, Cortez, & Bennet, 2018). These cases present compelling evidence for the use of lesson study on mathematical modeling to support teachers' conceptions of teaching mathematical modeling. Further research is needed to understand how lesson study on mathematical modeling can be employed in different contexts to achieve improvements in teachers' content and pedagogical knowledge with respect to modeling.

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