EXAMINING IN-SERVICE TEACHERS' DIAGNOSTIC COMPETENCE

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Teachers' diagnostic competence is essential for effective mathematics instruction. Prior studies have examined teachers' diagnostic competence using various approaches, such as asking teachers to assess students' erroneous work or anticipate potential learning difficulties. Few studies have examined how teachers interpret the significance of student errors, that is, to what extent the teachers think the flaws in students' work indicate a serious conceptual error or a trivial mistake that can be easily remediated. In this paper, we investigated the diagnostic competence of 2527 elementary in-service teachers by asking them to categorize errors in students' authentic place value errors in the context of decimal operations. Implications are discussed.

Keywords: Diagnostic Competence, Mathematical Error, Misconceptions, Decimal, Place Value

Perspectives

Teaching effectively and efficiently requires teachers to recognize 'what is' and 'how to' respond to the students' errors (Hill et al., 2008). Researchers (e.g., Artelt & Rausch, 2014; Schrader 2009; Südkamp et al., 2012) have defined diagnostic competence as the ability to anticipate or evaluate how well students perform on tasks. Diagnostic competence has been identified as a foundation of teaching expertise for decades (Weinert et al., 1990). Previous studies have worked on the conceptualization and measurement of teachers' diagnostic competence (Klug et al., 2013) as well as exploring how it affects students' learning (Guruzhapov et al., 2019; Helmke & Schrader, 1987). Researchers have assessed teachers' diagnostic competence by examining teachers' ability to analyze and identify errors in the students' work, anticipate common errors, and estimate the difficulty level of given tasks in order (e.g., Ostermann et al., 2018).

The present study is designed on the premise of defining teachers' diagnostic competence as how they infer the significance in the student errors. Such competence is important as it bridges teachers' diagnostic thinking of interpreting student work and making corresponding instructional decisions (Loibl et al., 2020). For example, while working on a multiplication problem such as 15 times 0.6, if a teacher considers a student response of 90 to be a *minor error* (e.g., a procedural error that misses the decimal point), he or she may respond by reminding students to add the decimal point after obtaining the solution of the mathematical operation. On the other hand, a teacher who regards this response as a *major error* (e.g., a conceptual error indicates a limited understanding of place value), might lead to a substantial intervention focused on the significance of the decimal point and place value. That is, perceiving an error as a *major error* more likely leads to conceptual instead of procedural remediation. This said we defined major and minor errors using the following text in the survey: "*Major errors* indicate a misunderstanding of key ideas that may persist even after sustained follow-up instruction; whereas *Minor errors* indicate a lack of awareness or inattention that can be addressed with brief follow-up instruction."

Markovits and Even (1999) reported a range of teachers' diverse interpretations and responses to instructional situations involving a decimal point. The data helped us in gaining an initial understanding of teachers' diagnostic competence on their knowledge related to decimal topics. To explore teachers' diagnoses of the significance of student errors more broadly, we focused on teachers' views concerning typical errors related to arithmetic operation with decimal notation. This research study was narrowly focused to better understand one case of teachers' diagnostic

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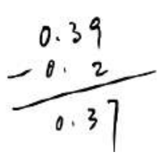
competency by answering the following research question: *How do in-service elementary teachers differ in their diagnosis of the significance of student errors with decimal place value?*

Method

The data for this study is drawn from a large-scale assessment focusing on evaluating teachers' knowledge for teaching rational numbers. For this paper, we included data from two items about teachers' diagnostic competence on decimal topics (see Figure 1). Both items are related to fourth-grade common core standards. Item 1 involves decimal subtraction (0.39 – 0.2 = 0.37) and Item 2 involves multiplication of a decimal and a whole number (15 × 0.6 = 90). The errors in both the items relate to the placement of decimal points. This error can be understood from two perspectives. If the error is understood to be procedural, it reflects students' missteps in completing an algorithm. In this case, the teacher may address the error by reminding the student about the correct steps of the algorithm. On the other hand, if the error is conceptual, it is reflective of students' inadequate understanding of place value. A teacher who considers the error as evidence of a misunderstanding, she or he may respond with more extensive instruction that is aimed at developing student understanding.

In this study, we tried to gain a better picture of how teachers interpret students' erroneous work. We provided definitions for the examinees of the two categories described above as major and minor errors, respectively. We then asked teachers to select the best option to complete the statement about each sample of student work, "In this student work sample, error or imprecision is (a) major and related to this topic, (b) minor and related to this topic, (c) related to a different topic, or (d) not evident." (Figure 1). Teachers who selected option (c) were asked to provide text to explain their reasoning.

Item 1



Considering the topic of place value, select the best option to complete the statement.

Item 2

Considering the topic of multiplying whole numbers and decimals, select the best option to complete the statement.

Figure 1. Two Item Samples

To answer the research question, we report the distribution of responses for all the options across 2527 elementary in-service teachers. To further understand teachers' reasoning on these questions, we analyzed all teachers' textual responses to option (c) (For Item 1, 3 were blank, thus n = 42; for Item 2, 3 were blank, thus n=36; see Table 2 and 3) using open coding (Cresswell & Poth, 2017). The teachers' exhibiting similar mathematical or pedagogical reasoning were grouped within one theme. For example, a teacher selected (c) related to a different topic and typed "Need to follow the same number value (tenth and hundredth)" for Item 1 which was categorized into the theme *place value* in Table 2. These themes and associated findings were discussed and reconciled during weekly group meetings.

Results

Teachers' Perspectives on the Significance of Student Errors

We found that teachers hold different perspectives on the importance of the same student errors. For the error in Item 1, 59% of the teachers interpreted it as a major error while 36% of them perceived it as a minor error. For the error in Item 2, 35% of participants indicated this error as a major error and 60% consider this is a minor and related error. A small percentage of teachers did not recognize the error in Item 1 (4%) or Item 2 (3%). For each item, around 2% of teachers thought the error was related to a different topic and reported their judgments on which topic was involved textually.

Table 1: The Distribution of Teachers' Perceptions of the Two Tasks

Option	Item 1		Item 2	
	N	%	N	%
Major and related to this topic	1482	59%	878	35%
Minor and related to this topic	904	36%	1521	60%
Related to a different topic	45	2%	39	2%
Not evident	96	4%	68	3%
Missing data	0	0%	21	1%

These data indicate that elementary teachers were more likely to classify the decimal notation error in the decimal subtraction problem as a conceptual error than they were the error in the decimal multiplication problem. From analyzing teachers' open-ended responses, we found more evidence to support this argument. We noticed that 42 teachers offered textual responses and 25 (60%) of them said the error was about place value (Table 2). We interpreted these responses as evidence that these teachers viewed the error as conceptual. Although these 25 teachers were able to identify this is a related error, they did not decide whether this was a major or a minor error, which may indicate the challenges of evaluating the importance of the error for these teachers. For example, one teacher responded "this is related to the topic, but the student could use interventions in decimal place value. This intervention could be beneficial to show him that .2 = .20 helping him to better line up his decimal number." The teacher has shown an understanding of the student's error and even offered intervention to remediate it, but she or he did not make a judgment about the importance of the error.

Table 2: Themes from Open-ended Responses on Item 1 (Option (c))

Theme of Responses (n = 42)	Number of Teachers (%)
Place value	25 (60%)
Computation/difference/algorithm/subtraction/operations	9 (21%)
Decimal addition vs subtraction	5 (12%)
Others	3 (7%)

Nine participants thought the error in Item 1 is related to broader topics such as computation, difference/subtraction, algorithm, and operations, which suggests these teachers noticed the error but did not necessarily interpret it to be related to place value error (Table 2). Five teachers thought the students should use addition instead of subtraction to solve the original word problem. In the "Others" category, one teacher thought students should use 0.02, one teacher suggested the student did not understand which number to subtract.

The teachers who offer textual responses to Item 2 (n = 36) held a more diverse interpretation of the student's decimal multiplication error. Contrary to Item 1, the teachers' responses were less dominated with the *place value* for Item 2 (n = 10, 28%). Four participants attributed the error to a lack of number sense. A teacher wrote that "[the student] doesn't understand the concept of a decimal - if you start with 15 groups of a number less than 1, your answer can't be larger than 15". Another teacher argued that the error related to "understanding the reasonableness of the answer due to values. For example, about 1/2 of 15 couldn't possibly be 60." These teachers seemed to identify a conceptual reason for the error but did not identify and describe the student's specific error, which may inhibit students' understanding of decimal (Markovits & Even, 1999). Nine teachers (25%) identified the error as procedural, relating to "carrying the decimal" or "placing the decimal point." Ten teachers (28%) thought the error was related to broad topics, such as "multiplying with decimals", "decimal", or "multiplication." Three teachers responded with something else such as "multiplying decimals is not a fourth-grade level math operation", "6", and "money".

Table 3: Themes from Open-ended Responses on Item 2 (Option C)

Theme open responses (n = 36)	*Number* (%)
Place value	10 (28%)
Number sense	4 (11%)
Decimal point placement	9 (25%)
Decimal multiplication or computation	10 (28%)
Others (e.g., not a 4th grade topic)	3 (8%)

In brief, from the teachers' written responses, we gained confidence in the larger finding that more teachers tended to identify the error in Item 1 as a conceptual error while more teachers interpreted the error in Item 2 error as a procedural error.

Discussion

Through this study, we found that two errors involving decimal notation were viewed by teachers in substantially different ways, with far more teachers classifying an error in decimal subtraction as major than a related error in decimal multiplication. These findings suggest the need to explore teachers' diagnostic competence concerning students' errors, and in particular to see how teachers perceive the importance of the error in students' mathematical learning. Such exploration goes beyond simply noticing students' errors because it requires teachers to identify students' errors, locate the error within certain mathematical topics, and justify its significance while providing conceptual remediation. This aspect of teacher knowledge may be more predictive of teachers' instruction because it requires teachers to apply their knowledge of student thinking to instructional decisions.

One limitation of this study is the possibility that some teachers held different interpretations of the terms major or minor error as these terms are not commonly used, which may affect the percentages of each option but may hardly affect teachers' general perception of each item. Although we provided the definitions before each item, some teachers may not have understood them as we expected. Thus, this study calls for future qualitative exploration of how teachers understand the major and minor error, the rationale of their option selections, and how they normally deal with such student errors through interviewing teachers and observing their teaching. How teachers interpret errors involving the decimal point in different mathematical contexts may deepen our understanding of teachers' knowledge for teaching decimals, an area in which little is presently known (Takker & Subramaniam, 2019).

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