CONNECTING IDEAS AND GESTURING DURING WHOLE-CLASS DISCUSSIONS

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This study extends our understanding of teachers' use of gestures during mathematics instruction. In particular, I examined the relation between teachers' gesture and the kind of mathematical connection verbally identified during whole-class discussions. Analysis of video-recordings of two teachers implementing a common unit of instruction revealed, in general, the teachers were more likely to use pointing and writing gestures rather than depictive gestures to make mathematical connections or support connection-making. However, the teachers used gestures differently during discussions based on the kind of mathematical connections discussed. These differences included the use of more than one type of gesture for an entity in a connection and whether both entities of a connection co-occurred in speech and gesture.

Keywords: Communication; embodiment and gesture; instructional activities and practices

In their review of research on learning and teaching with understanding, Hiebert and Carpenter (1992) found that explicit attention to mathematical connections during instruction was generative for students' learning, promoted recall, and supported students to develop a positive disposition toward mathematics. Unfortunately, The Third International Mathematics and Science Study (TIMSS) 1999 Video Study revealed there were few opportunities for and practically no discussions of mathematical connections in US mathematics classrooms (Hiebert et al., 2003). Interestingly, teachers in higher, achieving countries, such as Japan, were more likely to not only discuss connections but also to use gesture while doing so (Richland, 2015). While there is a growing body of evidence that gestures are beneficial for student comprehension (c.f., Hostetter, 2011) and support students' contributions during a discussion (Alibali et al., 2019), it is unclear if there is any relationship between teachers' gestures and the specific kind of mathematical connections made during instruction beyond connecting representations. This paper describes how two teachers' gestures varied in relation to the kind of mathematical connection being discussed during whole-class instruction.

Theoretical Foundation and Constructs

Embodiment and Situative Perspectives on Gestures

Broadly defined, *gestures* are movements of the body, usually of the hands and arms, for the purpose of communicating, and they sometimes accompany speech (McNeill, 1992). To understand how and why teachers gesture during instruction, I draw on the theoretical perspectives from embodied and situated cognition. From an embodied perspective, gestures emerge from simulated actions or perceptual states (Hostetter & Alibali, 2019). For example, asking an individual to think about a cup is also likely to activate the mental actions needed to hold a cup and so the individual may produce a "cupping gesture" with one hand. However, individuals may produce gestures that do not have roots in simulated actions or perceptual states. For example, a teacher may point at a mathematical object so that students may follow the referent of her speech (e.g., Is this [points at an expression] the same as this [points to a second expression]?). From a situated perspective, gestures are a semiotic resource that support interaction by developing, refining, or clarifying ideas (Goodwin, 2000). For example, Keene, Rasmussen, and Stephan (2012) argued that a sequence of gestures between an instructor and students over a series of lesson supported students' understanding of equilibrium solutions.

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In this study, I followed Alibali et al. (2014) in distinguishing between depictive, pointing, and writing gestures. *Depictive gestures* are "gestures that portray aspects of semantic content directly, via hand shape or motion trajectory, either literally or metaphorically" (Alibali et al., 2014, p. 76). Depictive gestures align with an embodied perspective of gestures. *Pointing gestures* are "gestures that indicate objects, locations, or inscriptions, usually with an extended finger or hand" (Alibali et al., 2014, p. 76). *Writing gestures* are "writing or drawing actions that were integrated with speech in the way that hand gestures are typically integrated with speech but that were produced while holding a writing instrument (usually chalk or marker) and that involved writing to indicate or illustrate the content of the accompanying speech" (Alibali et al., 2014, p. 76). Pointing and writing gestures align with a situative perspective of gestures.

Mathematical Connections

M*athematical connections* are the discursive ways in which an individual or community makes or describes a relationship between two or more mathematical entities. Entity is meant to encompass ideas, concepts, objects, representations, procedures, or methods. An individual or community may make a mathematical connection in variety of ways such as *connecting through comparison* (e.g., $\sqrt{a^2 + b^2}$ is the same as $\sqrt{b^2 + a^2}$), *connecting through logical implication* (e.g., If two distinct lines have the same slope, then the lines are parallel), *connecting methods* (e.g., Using the Pythagorean theorem or the distance formula can be used to find the distance between two points), or *connecting specifics to generalities* (e.g. A 6-8-10 triangle is an example of a Pythagorean triple; Singletary, 2012).

Methods

Participants and Data Collection

Melissa and Robin (pseudonyms) were selected to be part of this study from a larger research project that followed a cohort of secondary mathematics teachers in their teacher preparation program. Melissa and Robin were white females in their early twenties. They co-planned and co-taught an advanced 9th grade coordinate algebra course together during their student-teaching. Course goals included leveraging algebra to deepen and extend students' understanding of geometry. The data included lesson materials from one unit of instruction and video-recordings of the enactment of those lessons in two different class periods. This included 8 instructional days with Melissa as the focus teacher and 6 instructional days with Robin as the focus teacher. Each lesson recording was approximately 70 minutes in duration.

Data Analysis

First, I transcribed all video recordings of the lessons and included screen captures of the teachers' gestures with a short description. Then, I reduced the data to episodes of whole-class discussions about content-related activity (e.g., discussing the solution to a mathematical task) and not the day-to-day operation of school (e.g., checking attendance). From the reduced data, I then coded for connecting-periods (i.e., moments in whole-class discussions when a student or teacher made a mathematical connection). I will call connecting-periods just periods for simplicity. I excluded any periods if the mathematical connection in the period had already been discussed previously. This exclusion was done because Alibali et al. (2014) found that teachers were more likely to use gestures when the connection was novel to students. Next, using the Mathematical Connections Framework (Singletary, 2012), I coded the kind of mathematical connection expressed in the period. Finally, I coded and described the modalities (speech and/or gesture) used by the teacher for each entity in the connection.

Results

Across the lessons, there was a total of 60 periods. The teachers generally used at least one gesture (e.g., depicting, pointing, or writing) during a period (45 of 60). In about one-third of the periods, the teachers used two or more gestures to accompany speech about a mathematical connection (21 of 60). There were 13 periods when the teachers did not use any gestures and 2 periods where I was unable to determine if a teacher used a gesture due to the position of the camera (e.g., a teacher walked off camera).

The teachers' gestures differed depending on the kind of connection. For instance, they were more likely to use two or more types of gestures with speech to support discussions when connecting through comparison (13 of 23) or connecting methods (8 of 11). In contrast, there were few instances of a teacher gesturing with two or more types when connecting specifics to generalities (3 of 14). There were no instances of a teacher supporting discussions of connecting through logical implications using two or more types of gestures (0 of 11). In fact, it was somewhat common for connections through logical implication and connections of specifics to generalities to be unaccompanied by teachers' gestures (7 of 12 and 5 of 14, respectively).

Table 1. Modalities across kinds of connections

	Kind of mathematical connection			
	Comparison	Logical implication	Methods	Specifics to generalities
Two or more gestures with speech	13	0	8	3
At least one gesture with speech	21	4	11	9
Unable to determine	1	1	0	0
No gesture	1	7	0	5
Total	23	12	11	14

Furthermore, the teachers generally expressed both entities of a mathematical connection with gestures when connecting through comparison (14 of 23) and connecting methods (9 of 11) during instruction. In contrast, the teachers seldom expressed both entities when connecting through logical implication (2 of 12) and connecting specifics to generalities (3 of 14).

Table 2: Gesture use for entities within each kind of connection

	Kind of mathematical connection			
	Comparison	Logical implication	Methods	Specifics to generalities
One entity	7	2	2	6
Both entities	14	2	9	3
Neither entity	1	7	0	5
Unable to determine	1	1	0	0
Total	23	12	11	14

Lastly, for all kinds of connections, teachers often used pointing and writing gestures with speech over depictive gestures in relation to a single entity of a mathematical connection. This finding is in agreement with what Alibali et al. (2014) found. Therefore, there was no relation between the kind of

mathematical connection and the type of gesture. Table 3 outlines the type of gesture for at least one entity of a mathematical connection in relation to the kind of mathematical connection. Note that the sum for each kind of mathematical connection is different in Table 3 than the other tables because one entity of a mathematical connection could have been expressed multimodally (e.g., with a pointing and writing gesture).

Table 3. Type of gesture and kind of mathematical connection

Type of gesture (for at least one entity)	Kind of mathematical connection			
	Comparison	Logical implication	Methods	Specifics to generalities
Depictive	4	1	5	1
Pointing	16	1	8	4
Writing	14	3	7	8

Discussion

Novice teachers do gesture when discussing mathematical connections or supporting students' connection-making during instruction. This outcome is a distinctive shift from the TIMSS 1999 Video Study results and most likely reflects the recent emphasis on facilitating student-centered mathematical discussions in mathematics teacher education in the US. Novice teachers' use of gestures during discussions is also important because teachers' use of gestures has been found to lead to greater student comprehension (c.f., Hostetter, 2011) and promote students' contributions during a discussion (Alibali et al., 2019). Further, these novice teachers seldom used gestures when connecting through logical implication and connecting specifics to generalities. This is noteworthy because gestures are a semiotic resource for students' meaning making and a teacher's gestures may be a resource for moving students to a more productive meanings of logical implications (Hoyles & Küchemann, 2002) or to more sophisticated generalizations (Ellis, 2007). However, I do not argue that all the connections were productive for students or that gestures alone always lead students to develop productive meanings of connections. For example, Lobato et al. (2003) described how a teacher's use of ambiguous language of "goes up by" when describing the slope of a line along with her use of a sweeping gestures along one column in a table of values may have contributed to students' overgeneralization of slope as a difference rather than a ratio. One productive direction for future research is to determine if teachers are able to notice whether and how their own gestures are (not) productive for students' mathematical connection-making.

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