PUTTING THE "M" BACK INTO STEM: CONSIDERING HOW UNITS COORDINATION RELATES TO COMPUTATIONAL THINKING

Beth L. MacDonald
Utah State University
beth.macdonald@usu.edu

Colby Tofel-Grehl
Utah State University
colby.tg@usu.edu

Kristin A. Searle
Utah State University
kristin.searle@usu.edu

Andrea M. Hawkman
Utah State University
andrea.hawkman@usu.edu

Mario I. Suárez
Utah State University
mario.suarez@usu.edu

This theoretical commentary examines theory driven discussions in Science, Technology, Engineering, and Mathematics (STEM) fields and mathematics fields. Through this examination, the authors articulate particular parallels between spatial encoding strategy theory and units coordination theory. Finally, these parallel are considering pragmatically in the Elementary STEM Teaching Integrating Textiles and Computing Holistically (ESTITCH) curriculum where STEM and social studies topics are explored by elementary students. This commentary concludes with questions and particular directions our mathematics education field can progress when integrating mathematics in STEM fields.

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Computational thinking (CT) has recently been making a larger presence in elementary classrooms, yet it is still not yet clear how CT relates to young children's mathematical reasoning or even how it can be defined. Feldon (2019) explains "computational thinking has been characterized as a foundational competency, akin to reading and arithmetic" (p. 1). Given this characterization, the instructional technology field has yet to define CT (Feldon, 2019; Grover & Pea, 2013, 2018). Margulieux (2019) examined findings that suggest relationships between students' spatial reasoning and their Science, Technology, Engineering, and Mathematics (STEM) achievement when outlining particular theories that explain CT achievement. Pragmatic delineation of CT in the K-12 standards of the Computer Science Teachers Association (CTSA Task Force, 2011, p. 10) broadly characterize CT as a "problem-solving methodology" that draws from reasoning present in mathematics education, such as "abstraction, recursion, and iteration." These learning constructs and types of reasoning echo K-12 mathematics reasoning, effective mathematics practices, and mathematics learning objectives. Thus, *the purpose of this brief research report is to consider theoretically how CT reasoning (framed through spatial reasoning) relates to mathematics reasoning (framed through units construction and coordination).*

To frame this theoretical commentary, we first draw from spatial encoding strategy theory to explain how students' engagement with visual and mental representations may explain CT achievement. Second, we draw from the units coordination learning theory to determine how young children may be drawing from mathematics reasoning in elementary grade levels. From this theoretical framing, we consider particular parallels between these theories to determine multifaceted mathematics reasoning, as integrated in CT activities. Through an integrated STEM-driven curriculum grounded in social studies, titled, Elementary STEM Teaching Integrating Textiles and Computing Holistically (ESTITCH) (Hawkman et al., under review), we frame the pragmatic aspects of these integrated activities, and we delineate parallels between particular CT and mathematics reasoning, objectives, and practices. Moreover, we include social studies topics to determine how social artifacts leverage CT and mathematics engagement and what is gained with such an interdisciplinary instructional approach.

Theoretical Framework

This theoretical discussion is set in an emergent perspective paradigm (Cobb & Yackel, 1996), meaning we examine individuals' construction of mental objects and actions before considering the meaning gained through their engagement with social artifacts. Therefore, we begin by articulating theories framed with cognition learning science paradigms (Attkinson & Shiffrin, 1971; Baddeley, 1994; Clements & Sarama, 2019) and radical constructivist paradigms (Glasersfeld, 1995; Norton & Boyce, 2015), before drawing on this emergent perspective (Cobb & Yackel, 1996) within the context of STEM curricula. This framework begins by discussing spatial encoding strategy theory (cognition learning paradigm) before drawing from units construction and coordination learning theory (radical constructivist paradigm).

Spatial Encoding Strategy Theory

Through a review of the literature and drawing specifically from Parkinson and Cutts' (2018) findings, Margulieux (2019) proposed a spatial encoding strategy theory to explain the cognitive mechanisms related to individuals' spatial skills and STEM achievement. Margulieux explains that both the encoding of mental representations and the identification of landmarks (non-verbal representations) help individuals develop strategies and spatial skills (e.g., orientation, relations, and visualization). *Encoding* (making sense of) mental representations is best characterized in the cognition learning sciences where (1) individuals "chunk" information to act on in their working memory (limited memory capacity – Baddeley, 1994) and (2) individuals draw from attentional mechanisms (a component of executive functioning processes – Clements & Sarama, 2019) to determine what feature of a representation warrants attention (Attkinson & Shiffrin, 1971). For instance, when young children are asked to use text or symbols to solve problems in STEM fields (e.g., develop a code to move a LEGO® robot), they would need to map their anticipated results to a mental model that they can manipulate (Parkinson & Cutts, 2018). Prior to this experience, we argue young children would need physical experiences to form this model.

Moreover, Margulieux (2019) proposes individuals' mental representation construction partially depends upon individuals' development of non-verbal representations. For instance, when individuals chunk encoded information of mental representations, they are required to determine critical features and relationships of non-verbal representations (Margulieux, 2019). Thus, for individuals to encode mental representations successfully, they need to engage with/construct non-verbal representations that form these mental models.

Units Construction and Coordination Theory

Units coordination and construction refers to the number of levels and type of units children can construct and bring into a situation (Norton & Boyce, 2015). We utilize units construction and coordination learning theories to frame students' actions and establish transitions from their construction of *pre-numerical units* (physical material representing number) towards arithmetic units coordination. Children begin counting when first constructing pre-numerical units with which to use as material for future activity (Steffe & Cobb, 1988). These units are first constructed through children's external activity before becoming internalized (imagined activity) and then interiorized (automaticity).

To transition from pre-numerical units construction to arithmetic units coordination, children engage in one of four actions: unitizing, partitioning, iterating, and disembedding. Once students can count on they are next able to *unitize* (taking an item, or collection of items, as a whole unit that can be further acted upon) and *iterate* (making copies of a unit) units to construct number sequences (1, 2, 3, 4, 5, 6 …) (Norton, 2016; Steffe & Cobb, 1988). Once number sequences are constructed, students are able to *partition* (break into equally sized parts) these number sequences with which to count on from (Norton, 2016; Steffe & Cobb, 1988). To coordinate two levels of units, students would need to both iterate and partition (reversible actions), but would not yet be able to use them simultaneously (Norton, 2016). For instance, through counting, students could unitize two composite units (e.g., 3 and 12) where there are able to iterate three in a "count by" sequence (e.g., three, six, nine, twelve).

Once students coordinate all three levels of units and are able to do so in an anticipatory manner, they compose reversible actions and develop what Piaget (1970) described as logico-mathematical actions (operations). These operations allow students to construct number as a mental object with which to *disembed* a whole into parts while remaining cognizant of the whole (i.e., 12 is understood as 4 sets of 3) (Norton, 2016; Steffe & Cobb, 1988).

The CTSA (2011) articulate objectives grounded in some of these actions "abstraction, recursion, and iteration" (p. 10). For instance, as children iterate units, they construct sequences and are more readily able to abstract these sequences. Moreover, through children's composition of reversible actions (e.g., iterating and partitioning), they are able to recursively make sense of activity in STEM fields, providing them strategy development for future success.

Intersection of CT and Mathematics Reasoning within Social Studies Activities

Much of the mathematics education literature (Sarama & Clements, 2009) has found relationships between young children's spatial reasoning and mathematics development. By considering Margulieux's (2019) spatial encoding strategy theory, we argue that children's "chunking" of features from representations occurs in CT and in mathematics activities. By setting these activities in Social Studies, we posit children are using social artifacts to determine what warrants attention, which provides culturally responsive learning opportunities. Thus, we first consider parallels between one of the two CT learning objectives (see table 1) before considering how these might evidence themselves in the ESTITCH curriculum where integration of social studies and STEM provide meaning to students' units coordination.

In table 1, we outline relationships between two CT learning objectives and how they relate to corresponding elementary mathematics objectives and practices. For instance, when considering children's ability to decompose systems of computational thinking tasks, we propose their reasoning would be similar when they apply properties of operations, generate patterns, and evaluate expressions. To meet both sets of objectives, we posit they would need reason abstractly and attend to precision. In particular, students would be required to have two or three levels of interiorized units (dependent on type of operation) and would be required to determine critical features of a visual representation that relates to the goal of the task.

On day five, part 2 in the ESTITCH curriculum, students use stories centered on immigration, migration, and forced relocation to determine what landmarks are present in their own histories and how might they be used to form a timeline. Through their timeline development, they create circuits coded to represent these landmarks and proportional length/time to represent relationships between these landmark events. Through these activities, students are representing time and length in a scaled model, which presses them to generalize particular patterns abstractly and attend to precision of these events. Moreover, students are constructing units based on features of cultural artifacts they value to coordinate in a linear format. These integrated activities are powerful because they draw from cultural artifacts that children can connect to their mental representations of experiences and development of relationships between units that form these relationships.

Table 1: Intersection of Computational Thinking, Mathematics Standards and Mathematical Practices

Computational Thinking	*Operations and Algebra*	*Mathematical Practices*
Decomposition: Break down a task into minute details.	Apply properties of operations as strategies to multiply and divide (3.OA.B.5). Generate a pattern that follows a given rule. Identify features of the pattern not explicit in the rule itself (4.OA.C.5).	Reason abstractly and quantitatively (MP2) Attend to precision (MP6).
Pattern Generalization and Abstraction: Filter out information to solve a certain type of problem and generalize information.	Identify arithmetic patterns and explain them using properties of operations (3.OA.D.9). Write simple expressions, and interpret numerical expressions. Analyze patterns and relationships (5.OA.A.2).	Look for and make use of structure (MP7). Use appropriate tools strategically (MP5).

To emphasize the mathematics in this unit of study, an educator could have students construct visual models of decimals to represent time in such a proportional manner. For instance, if ten meter sticks represented one whole unit (one second), students could explore proportional relationships with smaller portions of a second with base-ten blocks (one centimeter in length) to explore coding with milliseconds. This type of precursor activity allows students opportunities to develop proportional relationships with physical models before requiring them to draw from mental models of the same relationships (MacDonald et al., 2018).

Conclusion

By considering the intersection of students’ reasoning associated with STEM and mathematics fields, we are more able to emphasize mathematical reasoning in curricula development while utilizing theories that focus on students’ mathematics reasoning. Moreover, as theory and associated curricula begins to emerge in the STEM fields, more questions surrounding theory and curricula need to be considered. For instance, how do STEM activities afford and/or constrain students’ mathematics reasoning? What trajectories in STEM are present for young children in prekindergarten classrooms as they transition to elementary classrooms? How might students with particular learning disabilities evidence STEM reasoning development in elementary classrooms and how might this relate to their access to mathematics in schools? Only through such multi-faceted theoretical frameworks and questions will our field continue to progress in a technology-driven society.

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