USING DESIGN-BASED TASKS TO TEACH AREA MEASUREMENT

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Geometric measurement is a critical domain that is difficult for many students. The focus of this study was to determine if the incorporation of design processes into instructional activities for area measurement may enhance engagement and learning of students from low-resource, historically marginalized communities. We adapted activities from a learning trajectory for area measurement, prompting Grade 3 students to integrate knowledge of arrays, multiplication, and area measurement. Results suggest the design focus prompted students' integration of knowledge of space and number by engaging in novel representations of designed objects and by prompting multiplicative thinking.

Keywords: Design Experiment, Elementary School Education, Geometry and Geometrical and Spatial Thinking, Learning Trajectories

We employed design and measurement tasks to teach mathematics across complex cultural contexts. Bishop (1988) considered mathematics a poly-cultural activity. He said people in many cultures engage in six fundamental mathematical activities to develop mathematical knowledge: counting, locating, measuring, designing, playing and explaining. Similarly, "…science learning can be understood as a cultural accomplishment" (National Research Council [NRC], 2012, p. 283). The Common Core Standards for Mathematics recommend students use geometry to solve design problems (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). The Next Generation of Science Standards (NGSS Lead States, 2013) suggest constructing explanations and designing solutions to prepare students for STEM fields. At Grades 3 – 5, students should, "Generate and compare multiple possible solutions to a problem based on how well each is likely to meet the criteria and constraints of the problem" (NGSS Lead States, 2013, p.46). We explored design work as a means to engage mathematical learning across cultural groups among students.

We selected area measurement and design problems as tools to support mathematics instruction across cultural contexts. We used a learning trajectory (LT) for area measurement (Barrett, Clements, & Sarama, 2017) to develop plausible assessment and instructional tasks (Barrett, Cullen, Behnke & Klanderman, 2017; Barrett & Battista, 2014; Battista, 2006, 2012; Sarama & Clements, 2009). The Cognitively-Guided Instruction group (Carpenter & Fennema, 1992; Fennema, Carpenter, & Franke, 1997) benefited from productive adaptations of tasks (Brown & Campione, 1996). Likewise, we drew on students' community funds of knowledge to adapt our tasks (Celedòn-Pattichis et al., 2018; Wager & Carpenter, 2012).

We had two goals: (a) determine if using design work to adapt instructional activities from an LT for area measurement would enhance learning and engagement, and (b) find whether design processes support mathematical learning. We expected the tasks to help students establish area units as cognitive tools for measuring space, through multiplication or addition. By anticipating spatial collections of units, students might extend skip counting and transition toward multiplicative reasoning in an array structure. We sought to promote students' use of arrays as models to measure area. We expect to suggest a model for improving the development of asset-based LTs that bridge cultural, community-based practices among elementary students. This was our rationale for adapting existing LT instructional tasks to (1) feature design processes, and, (2) integrate multiplication operations, arrays and area measurement.

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Method

Participants were a convenience sample of twenty-two Grade 3 students in an urban Midwest classroom and their teacher. Their school district consists of approximately 13,000 students (20.1% White, 57.7% Black, 11.3% Hispanic). Approximately 68% of the students at the school receive free or reduced lunch and 8% are English learners. We used a written assessment adapted from the LT (Barrett, Clements, & Sarama, 2017, pp. 105-115), with classroom observation to identify four levels of thinking among 22 students: Physical Coverer and Counter (3 students), Complete Coverer and Counter (5), Area Unit Relater and Repeater (9) and Initial Composite Structurer (3). We targeted these levels of thinking in design work within area measurement tasks.

Instruction Design Cycle

What we report here is the feasibility study phase of a design experiment (Middleton, Gorard, Taylor & Bannan-Ritland, 2008). This phase is meant to evaluate an intervention through qualitative methods such as observations, interviews, and case studies to determine what aspects of the intervention work and those that need improvement.

Prior to instruction, the researchers observed and helped students in the classroom to build familiarity and rapport. Later, we interviewed students in focus groups. We asked them how they may already use mathematics outside of the classroom to count, locate, measure, design, play or explain (Bishop, 1988). We conducted three lessons during one week of school in the Fall of 2019. Each lesson was led by one of the authors, with assistance from the classroom teacher. Each lesson began with whole-class discussion of a complex measurement question on area. The first lesson was an adaptation of patio tasks targeting LT levels often found among Grade 3 students (Barrett et al., 2017, p. 133-137). We set a designing task using a novel problem, to find the number of buses that could fit in a parking lot. Day 2, we asked them to design and draw a parking lot to fit a given number of cars or buses. Finally, on Day 3, we asked students to design and draw a park for pets, to provide room for a given number of dogs to move around freely for exercise. The teacher and researchers surveyed students' progress by assisting students who asked questions and posing questions to students while they worked. Students worked independently at first, and later in teams of two or four. The researchers kept field notes. Student work was collected for analysis. At the end of each lesson, the researchers reflected on what occurred in the class to develop the goal and a focal task for the subsequent lesson.

We analyzed the students' work in two ways. First, we examined all three tasks to determine strategies used to solve each. We asked ourselves how students made use of arrays or units in developing solutions. For the second and third tasks, we examined whether the students met the constraints of the task in the process of designing a solution.

Results

For the sake of the paper, we only discuss the results of the third day of instruction. We presented students with an image of a dog kennel. In the image it showed that a 6 x 6 foot square was an adequate area for a dog to run around in. We mapped out the square area on the floor of the classroom so students could see the space, walking around inside the mapped-out region to show the space needed per dog. We gave students a 1.5 x1.5 inch square cutout piece of cardboard. We told the students it represented the space that one dog needed to move around. The task we posed for them was to design a rectangular dog park (Constraint 1) that had enough room for 24 dogs (Constraint 2). At the end of the lesson, we had students present their designs. Table 1 shows the strategies students used in creating their dog parks.

Table 1: Strategies Used in Designing Dog Park

Strategies	Number of Students
Consistent Units with Some Use of Arrays	3
Consistent Units with Grouping	7
Consistent Units with no Grouping	7
Inconsistent Units No Clear Spatial Arrangement	1
No Use of Units Shown	4

Ten students made use of grouping and arrays to create their dog parks. Most students (n = 17) made use of consistent units in their designs. In dealing with the constraints students sometimes met both constraints (see Table 2), but still they were not successful in the total design project. For example, some students created a rectangular area that had room for more than 24 dogs. Other students accounted for the space for 24 dogs but appointed another rectangle to be the actual dog park.

Table 2: Met Dog Park Constraints

Constraints	Number of Students
Area of Park Design for 24 Dogs	6
Dog Park Rectangle	4
Both Constraints Met	6
Neither Constraint Met	6

Conclusion

Given the brief span of the intervention we conducted, we were not expecting students to move on to a new level of the learning trajectory (LT) for area measurement strategies. Rather, we used the LT levels as a rubric to find a suitable instructional level given students' exhibited knowledge of area measurement. Our findings with these design-focused tasks suggest students were creating designs and engaging with area measurement tasks that involved multiplication schemes in productive ways which is in keeping with AURR levels. This finding suggests design-centered tasks of this type offer ways of supporting student thinking and of observing their reasoning at these particular levels of a LT for area. This may provide a way of improving the instructional task descriptions as the LT is modified to broaden its impact on a wider range of students in various contexts and communities.

Furthermore, the design process of instruction appears helpful in focusing students' attention on the meaningful association among arrays, multiplication operations with number, and the measure of rectangular shapes. By engaging contexts that fit with our observations about the students' own experiences, we apparently gained access to familiar stories from their daily routines and community-based language for spatial quantity. The teacher was pleased to note that several students who typically did not engage in mathematics stayed engaged with the tasks for as long as they did. More work is needed to find what motivated this level of investment in the tasks.

Dealing with design constraints had mixed results from our vantage point; some students did not address any of the constraints, although other students successfully addressed one or more constraints. Nevertheless, students in Grade 3 demonstrated the capability to address design constraints related to measurement and space. The design emphasis, with the integration of measurement, multiplication schemes, and arrays as tools appears to be a viable way to adapt learning trajectory-based activities for area measurement. The lesson outcomes indicate promise that students in Grade 3 can engage in design activities with constraints related to multiplicative reasoning using skip-counting and grouping schemes. The interaction among these schemes may have prompted students to engage in the quantitative reasoning by access to their knowledge of such contexts as scanning to find whether a parking lot is empty, partly filled, or full. We believe the prominence and meaningfulness of the context provided a way for the mathematics of area measurement to be addressed as an integrated part of instruction on multiplication and arrays. This is consistent with work in statistics education showing the importance of linking context knowledge to the statistical schemes for organizing and reporting on data in such a context (Langrall, Nisbet, & Mooney, 2006).

We believe further design cycles may need to draw out a more comprehensive analysis of the multiplication processes and the arrays as tools for measuring the capacity of a parking lot to hold cars. We plan further work with the same students to have them redesign a dog park to meet constraints related to the area measurement and to the shape of the region (by requiring a rectangle). We also expect to include further ways to prompt students to check their own design by using a grouping scheme for collections of units. This could focus them more on iterating squares to fill space, and link to arrays and measuring area. The process of testing, designing, retesting and redesigning are vital STEM skills for students to develop (https://stem.getintoenergy.com/stem-skills-list/). The redesign process is important as we learn to extend the instructional tasks found in learning trajectories (e.g., the area LT) to different communities.

Ideally, teachers will use similar design-based tasks to adapt and work with their students in different community contexts. The principles of designing, measuring and describing, taken from analyses across a wide range of culture and communities by Bishop (1988) may productively inform both teachers and researchers who want to adapt learning trajectories for other content areas. Our findings suggest that designing, describing and measuring may be productive ways of engaging students as young as grade 3 in substantive mathematical projects. This may support them as they learn the structural advantages of noticing or setting up arrays to support area measurements and multiplication operations.

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