SECONDARY TEACHERS' DIFFERING VIEWS ON WHO SHOULD LEARN PROVING AND WHY

DIFERENTES OPINIONES DE MAESTROS DE PREPARATORIA SOBRE QUIÉN DEBE APRENDER DEMOSTRACIÓN Y POR QUÉ

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Reasoning-and-proving is viewed by many scholars to be a crucial part of students' mathematical experiences in secondary school. There is scholarly debate, however, about the necessity of formal proving. In this study, we investigated the notion of "proof for all" from the perspective of secondary mathematics teachers and we analyzed, using the framework of practical rationality, the justifications they gave for whether or not all students should learn proof. Based on interviews with twenty-one secondary teachers from a socioeconomically-diverse set of schools, we found that teachers do not share the same opinion on who should learn proving but they expressed obligations toward individual student learning as justifications both for teaching proving to all students and for not teaching proving to some students.

Keywords: Reasoning and Proof; Teacher Beliefs; High School Education; Equity and Diversity.

Reasoning-and-proving, the broad mathematical practice of conjecturing, justifying, critiquing arguments, constructing proofs and more (Stylianides, 2008), is central to the discipline of mathematics and can also be a powerful process through which students learn mathematics (de Villiers, 1995; Stylianides et al., 2017). Policymakers (National Governors Association & Council of Chief State School Officers, 2010; Secretaría de Educación Pública, 2014) and scholars (e.g., Mariotti, 2006) alike have called for reasoning-and-proving to be a part of all students' learning experiences in school. But there are also critiques of this general framing of learning "for all" such as Martin (2003) who pointed out that "for all" often comes as impositions on underserved groups, and Battey (2019) pointed out that "for all" can gloss over learners' individuality, proposing "for each and every" as a replacement framing. With regard to formal proof in particular, Weber (2015) noted that it may be unnecessary at the secondary level to explicitly develop "proving" and that it may be sufficient to push for clear explanations and valid justifications and that doing so may more easily integrate with students' mathematical experiences prior to secondary school.

Where do mathematics teachers, as the ones directly responsible for enacting curricular recommendations, stand on this issue of "proof for all"? How are teachers thinking about the scope and appropriateness of proof for students? Past studies have examined teachers' views of proof (e.g., Ko, 2010) or their views on mathematical processes including proof (e.g., Sanchez et al., 2015) but the question of who they think should learn proof is fundamental. In this study, we interviewed 21 secondary mathematics teachers from an economically-diverse set of schools in Cape Town, South Africa. Although outside North America, it has similarities to North American contexts in terms of mathematics teaching being heavily influenced by European colonization and having typical instruction that is procedural in nature (Webb & Roberts, 2017). Moreover, the question of who should experience proof is one with worldwide relevance as we consider broadly students' mathematical experiences.

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Personal and Theoretical Perspectives

Because this study involves our analysis of teachers' perspectives on proving, it is important that we reveal salient information about our own perspectives for the sake of transparency. Samuel is an American white man of Western European descent who attended rural public schools and then public universities where he earned degrees in both mathematics and education. Mitchelle is an African woman who attended the Kenyan elementary, secondary, and undergraduate education system, and is currently in the U.S. pursuing a doctoral degree. Rajendran, an Indian born in South Africa, attended urban primary and secondary public schools during the apartheid era and then proceeded to study at public universities where he earned degrees in both mathematics and education. Although from diverse backgrounds, we all share a view that proving—in the sense of constructing reasonably complete and logically valid arguments for mathematical claims—is important for all students in the general education system as well as most students in the special education system. Although this is our opinion, we value hearing the voices of teachers and taking seriously their perceptions of what is possible and why.

In terms of our approach to teachers' perceptions, we see teachers as participants in a cultural practice of teaching governed by norms (i.e., tacitly expected behaviors or unquestioned historical practices) and obligations (i.e., requirements perceived as inherent to their role as a mathematics teacher) (Herbst & Chazan, 2011). These norms and obligations influence the choices that teachers make in their own teaching (e.g., Webel & Platt, 2015). Obligations, in particular, can be used to categorize the justifications that teachers provide for their instructional choices. For example, a teacher may decide to present a proof to students rather than have them construct the proof independently because she feels an obligation to complete the lesson in a single class period and stay "on pace." Or a teacher may decide to emphasize formal terminology in a proof because he feels an obligation to the mathematics discipline to maintain "rigor."

We have two central research questions. RQ1) According to secondary mathematics teachers, who should learn proving in their formal mathematics education? RQ2) What justifications do secondary mathematics teachers provide for their answer about who should learn proving?

Method

The study was conducted in the Cape Town metropolitan area of South Africa, which is a port city on the southwest coast. South Africa, since its democratization in the 1990s, has pursued curricular reforms centered on universal education (Webb & Roberts, 2017). Its official standards call for elements of reasoning-and-proving to be taught to all learners. The 21 teachers participating in this study varied in their professional preparation and experience (from 1 year to 15 years teaching) but they all were mainly involved in teaching mathematics to grades 10–12 leaners. Their five schools were in drastically different socio-economic neighborhoods.

The first author, sometimes with the third author, conducted two types of semi-structured interviews. All 21 teachers participated in focus group interviews (approx. 20–40 minutes), organized by school, focused on the purposes of mathematics education and curricular issues related to proving. Ten of the 21 teachers also participated in individual interviews (approx. 10– 30 minutes) focusing on proving tasks and their experiences with proof learners. The analysis reported here specifically addresses the question of "who should learn proof in school?"

The interviews were transcribed and coding was in two phases. Phase 1 involved reading the transcripts and applying broad codes to any segments that related to the overarching research questions. We noted the groups of learners that teachers identified as who should learn proving. Phase 2 involved qualitative coding based on the practical rationality framework (Herbst & Chazan, 2011), particularly the professional obligations. We briefly describe these codes here:

- *Disciplinary*: teachers' perceived obligations related to mathematics as a subject area (e.g., proving is the "core" of mathematics)
- *Institutional*: teachers' perceived obligations related to the educational system, school policies, or administrators (e.g., proving is included on official assessments)
- *Individual*: teachers' perceived obligations related meeting the needs and expectations of specific learners (e.g., proving can help learners gain deeper understanding)
- *Interpersonal*: teachers' perceived obligations to balance the needs of a diverse class of learners and managing productive interactions (e.g., proving promotes respectful critique)
- *Worldly obligations*: teachers' perceived obligations related to the real-world usefulness of what is being taught (e.g., proving will help learners use logic beyond mathematics)

The *worldly obligation* code emerged from our own data set. Overall, multiple authors coded the obligations and met regularly to clarify (e.g., code two obligations within the same justification statement) and reconcile any discrepancies in the coding.

Findings

Several of the teachers expressed the opinion that some students should be exempted from the opportunity to learn proving (see below), but more than twice as many teachers expressed that all students should have an opportunity to learn proving. Others (i.e., some who only participated in the focus group interviews) did not express an opinion on this question, but the sections below provide brief findings with regard to the rationality that the teachers exhibited.

Teachers' Rationality for All Students Learning Proving

Teachers who stated that all students should learn proving provided a variety of justifications for that position. The most common justification related to the teachers' obligation toward *individual* student learning. Teachers explained that proving can help students to understand mathematical content in deeper or more inter-connected ways. For example, Panyanga said:

> All of [the students] should know how we get to things, not just the application… I'll just make an example with Pythagoras' theorem, you find that they know how to use it but they don't really understand it properly [without proving it].

An implicit obligation here is for the teacher to support students in understanding mathematics "properly," not just execute applications, and proving is something that promotes an understanding of "how we get to things." A similar point was raised by another teacher, Rhyan, who said about proving opportunities, "You have to give people space to experience the idea" because this helps them to move beyond knowing just "that a parallelogram has opposite sides equal" to understanding how that result connects with other pieces of knowledge in geometry.

Other teachers, in justifying that all students should learn proving, looked beyond the classroom. Specifically, teachers expressed a *worldly obligation* by connecting mathematical proving to students' current or future lives beyond mathematics. For example, Portia said that

> …to prove something is not just a mathematical skill, it's a skill in logic, it's a skill in trying to figure out and to validate your arguments. And that is not confined to only mathematics… I think it is a valuable skill and I think that everybody has the ability to do it. You don't need to say, 'Okay, this is exclusively for those who score high marks [in mathematics].'

Shabeer made a similar point that "being able to prove something in geometry, it helps you even being able to prove it in something unrelated to geometry." We also viewed references to general "critical thinking" as part of this worldly obligation.

Beyond individual and worldly obligations, a few teachers cited disciplinary or institutional obligations. We turn now, however, to those who had different opinions altogether.

Teachers' Rationality for Not All Students Learning Proving

Although one teacher commented that, were it in her power, she would remove proof from the official curriculum, we focus in this section on the several teachers who stated that only certain groups of students should learn proof. The most frequent justification that teachers provided for this position had to do with their *individual obligation* to support or cater to students' needs. For example, when asked who should learn proof, Shannon said the following:

> I think our students must learn proofs but not all the students. Because … with a particular class I can prove a theorem or I can do a proof. But with another particular class I will see that if I want to prove this theorem, I will do more harm than good… For those learners whom I can say they may be average to above average, yes. If we say let us prove it and let us use it, they will acutely enjoy the proving and the using of the proof.

For Shannon, she wants to teach proof when the result, as she perceives it, is an enjoyment of learning and students who see the usefulness of the proof. She has identified these benefits as being attached to some "but not all" students. For the other students, she refers to proving as doing "more harm than good," which we interpret to refer to confusion and struggles that can occur when she teaches proving. Other teachers expressed a similar obligation to help students avoid struggle. One said that proofs can cause students to become "discouraged" and another mentioned that those who "shouldn't learn the proofs are those learners [who] at the beginning are struggling… it's not going to be worth it to learn the proofs," whereas "those learners who excel should be focusing on the proofs because it'll help them understand actually most of the math much better." In this excerpt, avoiding proof and teaching proof were both rooted in individual obligations to either help students avoid struggle or help them achieve understanding.

Another justification that teachers mentioned had to do not with students' struggles but with their future plans after secondary school. For example, Shabeer said that "there is a certain group of students [who] should learn proofs and it depends maybe on what that student plans to do when he's done with school." This life-after-school idea connects with the *worldly obligation* in our framework. A similar point, but with different underpinnings, came from Trevelyan who said that proof should be "for the other students who are going to do something with the mathematics… are going to go and study further with the mathematics… because that's the core of what mathematics is." In this instance, a *disciplinary obligation* is evident as proof is an essential part of mathematics and thus is relevant for those who will pursue higher mathematics.

Discussion

Our goal was to recognize teachers' rationality because this is important as we attempt to work collectively to improve proof learning. As scholars, it is not safe to presume that all teachers necessarily view proving as worthwhile for all students. Yet, our findings suggest that teachers on both sides of the issue were deeply attuned to individual obligations. Thus framing proof as a way to support individual learning, and in particular addressing the idea (not expressed by the teachers) that proving can be fruitful for "struggling" students may be a way to find common ground. Conversely, appeals to the disciplinary obligation of proving may inadvertently send the message that proving is only for students who are on a pathway to higher mathematics. The worldly obligation may be an opportunity to promote more universal opportunities for proof learning. Or in listening to some teachers we may, like Weber (2015), have to take seriously the notion that *formal* proof may not be the most productive approach to reasoning for all students.

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