PRE-SERVICE SECONDARY MATHEMATICS TEACHERS' EVOLUTION OF COMMUNALLY AGREED-ON CRITERIA FOR PROOF

Yi-Yin Ko
Indiana State University
Winnie.Ko@indstate.edu

Dalton Edgecomb
Indiana State university
dedgecomb@sycamores.indstate.edu

Nathan Kooi
Indiana State University
nkooi@sycamores.indstate.edu

Developing communally agreed-on criteria for proof in a mathematics classroom has been found to empower pre-service secondary mathematics teachers' (PSMTs') learning of proof. To date, we do not know how creating class-based criteria for proof throughout a semester-long course with a focus on secondary school mathematics can promote PSMTs' understanding of proof. In this paper, we reported PSMTs' evolution of what constitutes proof by comparing their initial and revised class-based criteria for proof and investigating their videotaped lessons and video transcripts. Results indicated that PSMTs perceived mathematical values and norms of what counts as proof in their mathematics classroom community as the semester progressed.

Keywords: Teacher Knowledge, Reasoning and Proof, Pre-Service Teacher Education, Instructional Activities and Practices

Despite the importance of proof in school mathematics, pre-service secondary mathematics teachers' difficulties with proof are well-documented (e.g., Ko & Knuth, 2013; Yee, Boyle, Ko, Bleiler-Baxter, 2018). One of the primary challenges pre-secondary school mathematics teachers (PSMTs) face in understanding proof is that they might not perceive or accept mathematical values and norms with respect to learning proof in their classroom community (Dawkins & Weber, 2017). To address this challenge, an emphasis on teaching and learning proof as a social, negotiated, and sense-making process has been found to promote PSMTs' understanding of what constitutes proof and to enhance their ability to evaluate and construct proofs (e.g., Yee et al., 2018). To date, we have only found that Bleiler, Ko, Yee, and Boyle (2015) explicitly shared how undergraduate students in a transition-to-proof course developed, revised, and polished their communal criteria for proof throughout the entire semester. Given that mathematics teachers' knowledge for teaching proof has an impact on their instructional practices (e.g., Bieda, 2010; Stylianou, Blanton, & Knuth, 2009), there is a need to provide insight into how PSMTs' communal criteria for writing mathematical proofs is evolved throughout the semester-long course with an emphasis on secondary school mathematics. More specifically, this study investigated what PSMTs' initial communally agreed-on criteria for proof were and how they evolved throughout the semester.

Theoretical Framework

In the mathematics community, mathematicians actively engage in social practices to negotiate their agreement on the validity of acceptable proofs in the mathematics community (Harel & Sowder 2007). Along with this view, Stylianides's (2007) definition of proof incorporates a focus on the set of statements (i.e., definitions, theorems), the appropriate forms of argumentations, and representations accepted and understood within a particular mathematical community, which shows the general case will be always true without exception. However, research has not adequately investigated how engaging in this type of mathematics classroom community can facilitate evolution of PSMTs' understanding of proof in a course with a focus on secondary school mathematics. To address this research gap, attention must be paid to PSMTs' communally agreed-on criteria for proof in such courses throughout the entire semester.

Methodology

This study was conducted in an elective course focused on proof in secondary school mathematics at a Midwest University in the United States. The instructor (first author) selected, modified, or developed mathematical tasks in the domains of algebra, geometry, and number theory for the course. These three content domains are all vital to and pervasive in secondary school mathematics. In order for PSMTs to take ownership of their communally agreed-on criteria for proof and promote their self-regulation in learning proof throughout the semester, the instructor designed the course following the principle of the before-during-after (BDA) proof instructional sequence (see Ko, Yee, Bleiler, & Boyle, 2016 for detailed information about this three-part proof lesson plan and implementation). There were nine undergraduate students enrolled in the course, and only one PSMT, who transferred from another four-year college, had not taken any of the required proof-intensives courses for his major. The primary sources of data for this paper were the PSMTs' class-developed lists of writing good proofs, as well as the video recordings and their transcripts. The videotaped sections were transcribed by a research assistant and were validated for their accuracy by either the second or the third author.

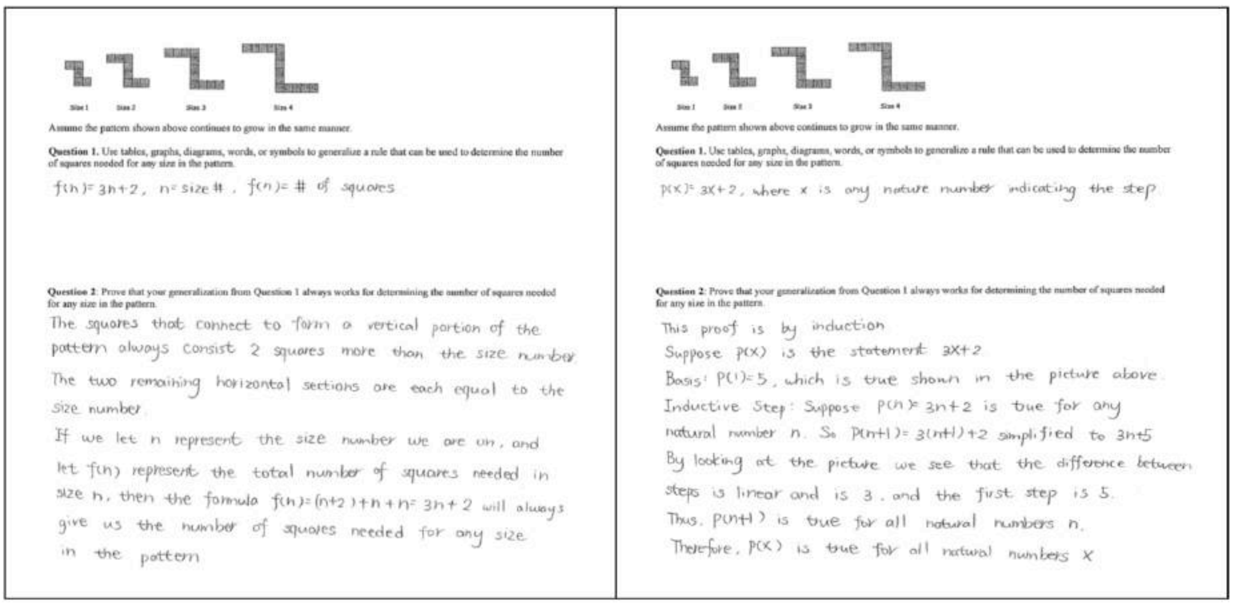

Figure 1: Two Sample Arguments for the Regina's Logo Problem

Results

Followed by the BDA instructional sequence, the PSMTs were asked to evaluate the validity of instructor-selected arguments (see the sample arguments of the Regina's Logo problem adopted from Seago, Mumme, and Branca, (2004) depicted in Figure 1). Also, the instructor served as the representative of the mathematics community to ensure the PSMTs' proposed characteristics for proof were acceptable. Then the whole class determined and ordered that generalization, logical order, correct terminology, clear and precise explanations, and identify given are the five most crucial characteristics of writing good proofs. The PSMTs then discussed their descriptions for each proof criterion as a whole class and came up with an initial list of writing proofs (see Figure 2).

Throughout the entire semester, the PSMTs had two opportunities to modify the initial list of writing good proofs. The first revision happened during the third week of the semester when the whole class did not come to a consensus on the validity of one instructor-chosen argument of the

Sticky Gum problem (Fendel, Resek, Alper, & Fraser, 1996) shown in Figure 3 according to their original proof writing rubrics. For example, Renee suggested that "we need to add something like correct and enough explanations for each step." For instance, Vivian said, "I was thinking maybe under [the] identifying the given [category]." Then the whole class agreed to change the description of the "Identify Given" category as "writing all given information that is pertinent to the proof." During the same time, some PSMTs also suggested that variables and symbols should be added to the "Correct Terminology" category (see Figure 2).

Proof Criterion and Its Order	Description of Each Proof Criterion		
	Initial List (Weeks 1-2)	First Revised List (Weeks 3-7)	Final List (Weeks 8-15)
1. Generalization	Have to be true for every scenario. No examples.	Have to be true for every scenario. No examples.	Have to be true for every scenario. No examples.
2. Logical Order	Progresses in a sensible manner. Each step needs to build from the previous step(s) and build to the conclusion.	Progresses in a sensible manner. Each step needs to build from the previous step(s) and build to the conclusion.	
2. Logical Order/Clear and Precise Explanations			Progresses in a sensible manner. Each step needs to build from the previous step(s) and build to the conclusion. Correct and sufficient explanations for each step. No unnecessary information.
3. Correct Terminology	Have to use the correct theorems, notation, rules, etc.	Have to use the correct theorems, notation, rules, variables, symbols, etc.	Have to use the correct theorems, notation, rules, variables, symbols, etc.
4. Correct and Precise Explanations	Correct explanations for each step.	Correct and sufficient explanations for each step. No unnecessary information.	
4. Assumption of Knowledge			Only assumed knowledge previously discussed/defined in our course.
5. Identify Given	Writing all the things that we are given at the start.	Writing all given information that is pertinent to the proof.	Writing all given information that is pertinent to the proof.

Figure 2. Initial and Evolution of the Communal Criteria for Proof.

> We know that we need to have k number of gumballs, where k represents the number of kids, so that each child has one. We also know that each child wants to have the same color. If we have c number of colors, then worst-case scenario would be that each time we use the machine, we receive a different color gumball. However, after c number of uses, we would have either multiple of at least one color or one of each. Therefore, if we only had one of each color, after the next repetition, we would have more than one in at one color. Following this pattern, we would need to keep using the machine as many times as there are colors multiplied by one less than the total number of kids. This would get us, in the worst-case scenario, one matching gumball for every child except the last one. In fact, if every turn of the machine produced a different color until all colors had been used and repeated this process continuously, we would, at this point, have enough of each color for every kid except one. We would, therefore, only need to use the machine one more time to find a match for the final child. From this information, we can extrapolate that c(k-1) represents the number of times we need to use the machine in order to have a full set of colors for all of the children except for one; adding one more turn would fulfill the request of the stingy children by giving us a full set of one color. Therefore, the expression c(k-1)+1 satisfies the conundrum for any number of children with any number of colors.

Figure 3: A Debated Argument for the Sticky Gum Problem.

As the semester progressed, PSMTs had another opportunity to revisit and modify the proof rubrics. When negotiating the validity of the sample arguments for the statements, "Suppose m, n, and p are positive integers. If m is a factor of n, and m is a factor of p, then m is a factor of $n + p$," some PSMTs pointed out that one of the biggest problems of the first revised proof criteria is clear and logical explanations. Given that not all the PSMTs in this class had completed the required proof-intensive courses for their major, they discussed how much information they should include in each argument to be considered as a proof based on their class rubrics. For example, Ethan explained that their group added the assumption of knowledge category because we "can't just assume that people know things that could have been learned in other classes or in other settings."

After the whole class discussion, all the PSMTs decided to keep the top four and the last characteristics of writing proofs and to combine the other two categories, "Logical Order" and "Clear and Precise Explanations," into one criterion. They also added the fourth criterion pointing out the importance of writing proofs that only used definitions, theorems, or principles that had learned, accepted, and discussed in this class (see Figure 1). The second revised list served as the final version of writing proofs, because all the PSMTs felt that this checklist was sufficient for them to construct and evaluate proofs for the rest of the semester.

Discussion

Throughout the semester the PSMTs did not make a major change on their initial list as seen in Figure 1, concurring with Bleiler et al.'s (2015) finding that instructor-selected sample arguments served as good foundations for students to consider the important characteristics of writing proofs. Another feature of the results is that the PSMTs' communal criteria for proof are consistent with mathematics professors' characteristics of a well-written proof, including logical correctness, clarity, and fluency (Moore, 2016). In addition, the PSMTs recognized that their written proofs should be readable and understood by audiences in their mathematics classroom community as the semester progressed. These two findings reveal that the PSMTs perceived mathematical values and norms of learning proof in their mathematics classroom community through constructing, evaluating, negotiating, and making sense of arguments, which more closely aligns with the practice of mathematicians (Harel & Sowder 2007). Even though PSMTs developed the initial list of writing good proofs and revisited it as the semester progressed, they still negotiated their evaluations for some of the instructor-selected arguments to be considered as proofs or not. Given that PSMTs' communal understanding of what counts as a proof is affected by their instructor of a proof course, comparing how PSMTs and their instructor use their class-developed criteria for proof to evaluate the same arguments can provide more insight into their individual interpretations of each proof criterion within their mathematics classroom community.

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