INSTRUCTIONAL TENSIONS FACED WHILE ENGAGING HIGH SCHOOL GEOMETRY STUDENTS IN SMP3 TASKS

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This study engaged HS geometry students in the reasoning-and-proving process through the use of novel tasks aligned with Standard for Mathematical Practice (SMP3) (construct viable arguments and critique the reasoning of others). The tasks facilitated opportunities for students to engage in SMP3 by (a) proposing a conjecture; (b) drafting an argument for their conjecture; (c) critiquing each other's arguments; and (d) revising their arguments based on peer feedback. In this study, we describe the instructional tensions that surfaced during the implementation of the tasks and the way the teacher addressed those tensions in her class (Berry, 2007). The two most common tensions were between action and intent when launching the tasks and between telling and growth during the draft and critique phases. Findings raised important questions of how to support students in learning what counts as a mathematical conjecture or critique.

Keywords: Instructional activities and practices, Reasoning and Proof, Instructional Vision

Introduction and Purpose

The Standards for Mathematical Practice (SMP) articulate eight domains of mathematical thinking students should gain expertise in across K-12 grades (National Governors Association [NGA] Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010). Specifically, SMP 3 states that students should "construct viable arguments and critique the reasoning of others." Historically, constructing formal deductive arguments (proofs) has been restricted to high school geometry courses (Herbst, 2002). Proof tasks in commonly used U.S. Geometry textbooks provide opportunities for students to engage in some aspects of SMP3, such as posing a conjecture, constructing a proof, investigating a statement, and developing a rationale (justification) for mathematical claims with varying degrees of frequency across categories (Otten, Gilbertson, Males, & Clark, 2014). In contrast, the textbooks analyzed provided relatively few opportunities for students to find a counterexample and did not explicitly ask students to respond to the reasoning of others or construct arguments with the goal of communicating to their peers. Although Otten and colleagues (2014) did not report the percentage of textbook exercises where students were asked to engage in multiple forms of reasoning-and-proving activity within the same task, the differences between categories suggests that students are not consistently engaging in the multifaceted process described in SMP 3.

The purpose of this study was to implement a series of novel tasks designed to engage high school geometry students in the reasoning-and-proving process (Stylianides, 2007) in alignment with the multifaceted approach described in SMP 3. Specifically, the tasks were novel in that students were asked to (a) propose and investigate their own conjecture instead of one provided for them; (b) critique each other's arguments; and (c) revise their argument based on peer feedback instead of teacher feedback. In this preliminary study, we analyze the instructional tensions (Berry, 2007) that arose when implementing the tasks. In doing so, we contribute greater insights into challenges that classroom teachers might face when navigating across classroom cultures, towards one that is centered around students' mathematical ideas instead of one based on ideas presented by the teacher or textbook.

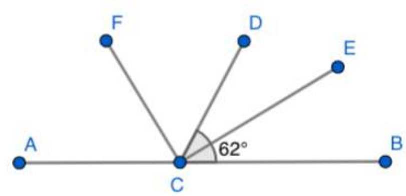
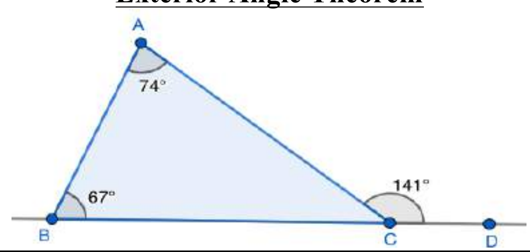
Theoretical Framework

Teacher's instructional decisions are shaped by their personal knowledge and beliefs as well as their obligations to a variety of stakeholders, including the mathematics discipline, individual students, interpersonal dynamics in the classroom, and their broader institutional context (Herbst & Balacheff, 2009; Herbst & Chazan, 2003). Tensional dilemmas, or tensions, surface when there is a contradiction between their beliefs, knowledge, and obligations such that there is no clear decision that adequately addresses all of their concerns (Lampert, 1985). Some teachers choose to prioritize one obligation over another, while other teachers, such as Lampert (1985), instead try to manage the tensions through instructional decisions that reduce the dilemma without completely resolving it. For example, Berry (2007) described six instructional tensions she experienced in her role as a teacher-educator: telling and growth; confidence and uncertainty; action and intent; safety and challenge; valuing and reconstructing experience; and planning and being responsive. Although instructional tensions have been described across multiple contexts (e.g., Berry, 2007; Herbst, 2003; Rouleau & Liljedahl, 2017, Webel & Platt, 2015), more research is needed with respect to teachers' experiences when enacting the Standards for Mathematical Practice (SMPs).

Methods

Instructional Sequence

The tasks used in this study were developed with the goal of engaging students in multiple facets of SMP3. Two of the tasks had been previously implemented with secondary students (Conner, 2018); the remaining three tasks were constructed using similar design principles. Each task allowed students to develop a conjecture about a geometric relationship involving an infinite class of objects. The diagonals of a parallelogram and classes of similar polygon tasks also allowed for students to pose and investigate multiple correct conjectures (see Figure 1).

Angle Bisectors of Linear Pair	**Exterior Angle Theorem**
 Given: $\overline{CE}$ bisects $\measuredangle BCD$; $\overline{CF}$ bisects $\measuredangle ACD$	

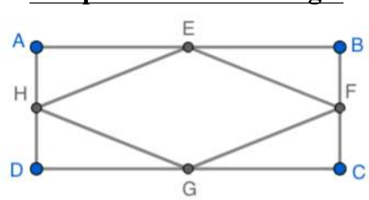
Diagonals of Parallelograms	**Midpoints of a Rectangle**	**Classes of Similar Polygons**
Draw a few parallelograms on your paper. Draw in the diagonals. Make a conjecture about the diagonals of all parallelograms.	 What quadrilateral is formed when you connect the midpoints of a rectangle?	After describing what it meant for all quadrilaterals to be similar, students were asked to conjecture which classes of polygons (e.g., squares) were all similar to one another.

Figure 1: SMP3 Tasks

The teacher launched each task by posing a scenario for students to consider through the use of a verbal description or a computer-generated representation. Students then formed an initial conjecture about the generalization of the relationship. Next, the teacher either discussed the individual/group conjectures with the class and had all students prove the same conjecture or students proved their own conjecture without a whole-class discussion. For example, students wrote a proof for their own conjectures about the diagonals of a parallelogram. Their conjectures included: diagonals are congruent, diagonals are perpendicular, diagonals create two pairs of congruent triangles across from one another, and diagonal intersect at each other's midpoint. Students worked individually or in small groups to develop a draft argument proving or disproving their conjecture. Once draft arguments were completed, students exchanged papers and provided feedback to one another. Students then drew on their initial arguments and peer feedback to complete a final draft of their argument. The task concluded with a whole-class discussion around how to prove one of the conjectures, which drew on ideas from students' written work. In instances where there were multiple student-conjectures, the remaining ones were discussed in class but not proven.

Context

The study took place in three geometry classes, all taught by the second author, located in a rural high school in the Midwest region of the United States. Proof-writing was a regular part of instruction, with proofs written weekly in class and, less frequently, assigned as homework. Each task was completed in 1 – 1.5 class periods (roughly 60 - 90 minutes).

Data and Analysis

Data for this study consists of a HS geometry teacher's oral reflections after each of the five tasks (see Figure 1). During the reflection process, the first author asked open-ended questions, such as "How do you think the task went?" and "What issues arose during the lesson?" Since the classes were not video recorded, the teacher consulted students' written work and was read portions of the researcher's field notes to help recall what happened.

Using Berry's (2007) framework, the researcher coded the teacher's reflections after each task for the instructional tensions that surfaced and then looked for themes across tasks. Next, the researcher qualitatively coded the reflections for instances where the teacher described how she navigated the identified tension during the lesson. In order to establish trustworthiness and reliability, the researcher and teacher conducted a member check on the themes and how she addressed the identified tensions in her teaching (Lincoln & Guba, 1985).

Findings

Action versus Intent

The teacher's goals (intent) was to provide students with opportunities to engage in different facets of SMP3 and improve their proof writing skills. Her goal for students to pose and investigate their own conjectures resulted in tensions regarding how to introduce the tasks in a way that did not undermine this goal. For instance, after noticing that students had relied on examples during a previous task, she described questioning how to introduce the diagonals of the parallelogram task in a way that would encourage students to generalize past specific examples.

> I was so hesitant. I didn't want to label the angles. And I didn't do one [diagram] as a class collectively. Trying to get them *again* to generalize past the examples, cause now that I had that experience with, 'oh, they just draw in examples'… how to word my language to try to get them to move that way initially, and not waiting until the revisement [discussion] period.

In this task, the teacher's actions at the beginning of the task did not undermine her goal to have students form conjectures. Instead, having students construct multiple examples and discuss their conjectures in small groups resulted in them realizing on their own when a conjecture was false.

During the exterior angle theorem, the teacher ultimately launched the task in a way that guided students towards the specific angle relationship, despite her goal of having students pose their own conjecture. The teacher initially told students to "make a conjecture about the exterior angle of a triangle and its interior angles". This resulted in the student conjecture that $m\measuredangle ACB$ and $m\measuredangle ACD$ added to 180°. Recognizing that their conjecture would not result in meaningful reasoning-and-proving activity, the teacher and researcher decided to guide students towards the anticipated angle relationship using a series of questions about what the students noticed in the diagram. "We had a purpose, so at that moment it was less about individual student and more about whole class so we could move forward" (Teacher).This tension between focusing on the intended mathematical content and allowing students to engage in the SMP3 process using their own conjectures was present throughout the lessons.

Telling versus Growth

Throughout the draft and critique phases of the lessons, the teacher experienced tensions between giving students direct feedback or guidance and allowing them to discover and improve their arguments on their own. For example, when a group's draft argument did not match their conjecture, the teacher struggled to make sure she was not saying "too much to them," hoping other groups would notice and provide that feedback. When a student asked if they could create a drawing to prove their conjecture, the teacher struggled to respond while also being "very conscientious of not saying that they were right or wrong." During the critique phase, the teacher felt like she had to encourage students to write down their questions and comments and "give them permission to be critical" when providing feedback to their peers.

The teacher used the whole class summary as a way of resolving prior tensions to directly address issues in students' work related to their justifications, precision in language, and generality of their arguments. She drew on students' ideas throughout the proof construction process to show she valued the thinking they did in the previous lesson phases.

> I remember trying to think of how to tie in what they were doing to what I was saying. So you guys used examples and this is how we go further. […] I remember trying to draw on what they did, so that it didn't seem like a waste of time.

Across the lessons, the teacher prioritized students' growth and voice during the beginning parts of the lesson. During the summary, she built on students' comments while also making sure the argument encompassed all cases and used mathematically precise language.

Implications

Teachers, often implicitly, navigate tensions throughout their lessons as a result of competing obligations that surface (e.g., Cohen, 1990; Herbst, 2003). In this study, the specific tensions surfaced in part due to a desire for students to have ownership in all stages of the tasks. When preparing teachers to incorporate SMP3 into their practice, it can be helpful to acknowledge these potential tensions and support teachers in reflecting on how they might navigate them in their class. Although the tensions experienced were not specific to the novel task used (see e.g., Rouleau & Liljedahl, 2017), the focus on SMP3 surfaced additional questions around how to support students in developing understanding of what counts as a mathematical conjecture or useful critique. Specifically, to what extent should teachers intervene when students pose conjectures that will limit their reasoning-and-proving opportunities (e.g., a conjecture that is a direct application of a definition)? What are ways teachers can support students in providing meaningful critiques? How can teachers balance the tension between developing students' understanding of proof and providing opportunities to engage in the different facets of SMP3?

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