RELATIONSHIPS WITH MATHEMATICS: THE IMPORTANCE OF AGENCY AND AUTHORITY

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Mathematics classrooms are spaces where teachers provide students with opportunities that will inevitably shape their conceptions of the subject and their own abilities to learn it. Therefore, it is important to understand how a classroom community defines mathematical knowledge, mathematical practice, and what it takes to be a person who is successful in mathematics. The study uses interviews with teachers and students in two classrooms plus a district wide survey to understand the relationships with mathematics they construct. The paper ends with a discussion of specific areas of pedagogy that could support the development of productive relationships with mathematics by more authentically centering student thinking in the classroom.

Keywords: Teacher Knowledge, Equity and Diversity

Introduction

For many, math is seen as requiring rote memorization and the regurgitation of procedures with little to no room for free thinking. Research shows that such approaches to math learning are related to low achievement (Boaler & Zoido, 2016, PISA, 2012, Gray & Tall, 1994). Mathematics classrooms are spaces where teachers provide students with opportunities that will inevitably shape their conceptions of the subject and their own abilities to learn it. Therefore, it is important to understand how a classroom community defines mathematical knowledge, mathematical practice, and what it takes to be a person who is successful in mathematics. This paper builds upon Boaler's 2002 framework for a relationship with mathematics in terms of knowledge, practice, and identity. The study uses interviews with teachers and students in two classrooms plus a district wide survey to understand the relationships with mathematics they construct. The paper ends with a discussion of specific areas of pedagogy that could support the development of productive relationships with mathematics by more authentically centering student thinking in the classroom. These include a shift towards open and project-based curriculum and an increase in value placed on student mistakes and struggle.

Literature Review

Aguirre, Mayfield-Ingram, and Martin (2013) define mathematics identity as "the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives" (p. 14). A student's mathematics identity will be formed in part by the ways they have been positioned in their particular learning context (Holland et. al, 1998). However, these conceptions of math identity are missing a key component to students' experience in the mathematics classroom-- the behaviors and practices they are expected to engage with while doing mathematics.

Teacher expectations of student mathematical behavior can be thought of through the lens of agency and authority. Agency refers to the extent to which students are able to express and use their own ideas in mathematical problem solving and authoring (Boaler, 2002). Student agency depends deeply upon the beliefs held by both teachers and students about what is expected of students in their role as problem solvers. This ranges from one end of a spectrum where the students are able to approach problem solving creatively, using their own ideas and methods to the opposite end where students are

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expected to use one specific procedure that's been told to them by another authority (i.e. the teacher or textbook). Part of this type of agency involves the extent to which students see this aspect of problem solving as being within their control. Gutstein (2007) found that when students experienced a strong sense of agency in their problem solving, they were empowered to interrogate knowledge sources and critically analyze material rather than simply receive it as truth. This sense of agency resulted in students deconstructing representations using mathematics to deepen their understandings of new material rather than searching for a pre-determined solution strategy.

Similar to the concept of student agency in the classroom is the concept of student authority. According to Cobb, Gresalfi, and Hodge (2009), authority in the mathematics classroom pertains to who decides what constitutes mathematical legitimacy. In some classrooms, this authority could lie solely with the teacher or textbook whereas in other classrooms it may be shared between the students, teacher, and textbook. Amit and Fried (2005) found that when the teacher is the main authority figure, students oftentimes use mathematical concepts introduced by the teacher unreflectively. In other words, students blindly reproduce what the teacher has shown without further thinking. These authors also found that this particular authority dynamic can hinder the productivity of collaborative learning efforts. For example, when students are working in groups, but the teacher is seen as the authority, little impetus exists for authentic collaborative problem solving.

Relationships with Mathematics

Boaler (2002) introduced the concept of a disciplinary relationship (see Figure 1). As knowledge, practice, and identity develop for a mathematics student, they each contribute to an overall relationship with mathematics. First, we consider the identity aspect of the relationship which is broken down into two parts: beliefs about one's role, and one's mindset. Within a mathematics learning setting, students will develop their own beliefs about what it means to learn math. A student may expect to be a passive receiver of knowledge, an active participant exercising agency and mathematical authority, or somewhere in between (Belenky, Clinchy, Goldberger, & Tarule, 1986). Additionally, they may believe that their mathematical abilities are static and fixed, or that they can be cultivated and grown (Dweck, 2005). Second, we consider the knowledge and practice aspects of the relationship with mathematics which together constitute the student's beliefs about the nature of mathematics and doing mathematics. The student may believe that knowledge in mathematics is made up of facts and procedures to be memorized, or they may see math as a web of ideas connected by logic and reasoning (Boaler & Zoido, 2016, PISA, 2012, Gray & Tall, 1994). Finally, the student may see the practice of doing mathematics as effortlessly and quickly understanding material or requiring struggle, learning from mistakes, and creative thinking (Boaler, 2015).

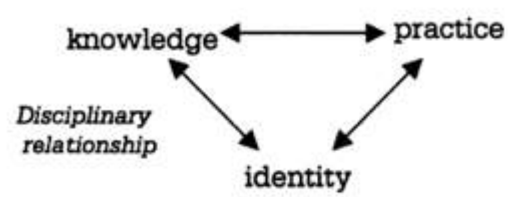

Figure 1. Framework for Disciplinary Relationships adapted from Boaler, 2002

Oftentimes, when a student has developed an unproductive relationship with mathematics, they respond to creative approaches in problem solving by stating something like, "am I allowed to do that?" As part of Boaler's 2002 study, she observed many classrooms referred to as "traditional" and "reform"- oriented. One of the most observable differences between these two types of classes were the role that agency played in each of them. Within the traditionally oriented classrooms, students were expected to follow standard procedures of the discipline. In these cases, the students' relationships with mathematics revolved around the agency and authority of the discipline. The students expected to follow the procedures and practices defined by the discipline.

In the reform-oriented classrooms students were "required to propose 'theories', critique each other's ideas, suggest the direction of mathematical problem solving, ask questions, and 'author' some of the mathematical methods and directions in the classroom" (Boaler, 2002, p.45). While it appeared that these classes offered more agency to students, Boaler clarifies that these students engaged with what Pickering (1995) calls the "dance of agency" between the established methods of the discipline and their own knowledge and practices (Boaler, 2002). (Note: Pickering (1995) found that professional mathematicians also engage with the "dance of agency" when developing and discovering new mathematics.) Through this "dance of agency" students would employ standard procedures coupled with their own ideas to adapt and extend methods in new and unknown contexts. The students developed mathematics relationships that gave them a sense of agency and allowed them some authority over their mathematical knowledge construction.

It is important to note that these students' opportunities to engage with the "dance of agency" were dependent on a number of things including the presence of an engaging project based curriculum but also the practices that teachers expected students to exhibit (Boaler, 2002). When it comes to mathematical practices (or ways of engaging with mathematics), the field provides what seems like a never-ending list of possible practices that teachers might focus their classroom towards (CCSS Mathematical Practices, 2010; NCTM Process Standards; Stipek et al., 1998). These practices include: communication, reasoning, proof, representations, justification, argument, sense making, and many more. While each of these practices are undoubtedly important, teachers cannot be expected to prioritize each one to the same extent so they must make choices based on their pedagogical beliefs or the needs of their students.

The decision of what to prioritize is complex in today's mathematics classrooms where students enter with an increasingly diverse range of strengths and needs. That is not to say that heterogenous classrooms are an issue. In fact, many research studies have shown that all students benefit from learning in de-tracked and heterogenous classrooms (Boaler, 2006 & 2011; Burris, Heubert, & Levin, 2006; Horn, 2008; Porter et al, 1994). However, knowing that the expectations for practice in the mathematics classroom will form the normative identity and ultimately influence the students' relationships with mathematics, it is imperative to know what teachers emphasize and how these priorities are taken up by students. This leads to the research question guiding this study: How are teachers and students constructing relationships with mathematics together?

Research Methods

Study Context and Participants

Although no two students will have the same relationship with mathematics, theoretically, the widest range of differences in these relationships would be apparent in a heterogeneous classroom where students represent a great range of prior experience and achievement levels. For this reason, this study focuses on heterogenous Algebra 1 classrooms.

The data for this study comes from a research project on a large urban school district in the Bay Area that had recently implemented a district-wide change of their mathematics course taking sequence by de-tracking mathematics through sophomore year and enrolling all 9th grade students in Algebra 1. Previous to this policy, students took algebra in eighth grade and 40% of students were failing and re-taking Algebra 1 (Hull Barnes & Torres, 2018). Since the implementation of the de-tracking policy, this failure rate has dropped to just 8% (Hull Barnes & Torres, 2018).

The student body of the district is culturally diverse with about 35% of students identifying as Asian, 27% Latinx, 15% White, 7% African American, 5% Filipino ,1% pacific islander, and <1%

American Indian (Facts, 2018). Furthermore, approximately 55% of students are considered Socioeconomically Disadvantaged, 29% of students are designated as Language Learners, and 11% are students diagnosed with Special Educational Needs (Facts, 2018). The students at Park High School represent an even more diverse community than that of the district with a higher percentage of socioeconomically disadvantaged students and greater percentage of students of color.

The participants in this study include two Algebra 1 teachers at Park High School and 6 of their students, for a total of 12 students. See Table 1 for more details about the participants.

Table 1: Participant Details

Teachers	Teacher Details	Student Pairs and designation[1]
Ms. Anderson	Early career teacher	Jackie and Kim (high achieving)
		Silvia and Arthur (turn around)
		Leta and Jose (low achieving)
Mr. Lang	Veteran teacher	Teresa and Mario (high achieving)
		Chantel and Steven (turn around)
		Marina and Lucy (low achieving)

Data

The data includes interviews with teachers and students in the two focal classrooms and results from a survey on mathematical mindset administered to 555 9th grade algebra students across the district. The interview questions for both teachers and students were developed using Rubin & Rubin (2008) as a guide. Teachers were interviewed during their prep periods and students were interviewed in pairs during class time.

The Mathematical Mindset Survey was developed by the youcubed research team and validated through previous research studies. The survey contains 27 questions with Likert scale answer options: Strongly Disagree, Disagree, Somewhat Disagree, Somewhat Agree, Agree, and Strongly Agree. The survey was conducted using the Qualtrics online software and offered in English, Spanish and Chinese. There are approximately 4,750 ninth grade students in the district, and the email requested teachers to give the survey to at least one of their 9th grade Algebra 1 classes. The distribution of the survey resulted in a total of 555 student responses to the survey representing a sample from 8 different high schools and 23 different teachers within the district.

Methods

Interview Data

To analyze this data, the researcher followed the analysis guidelines for inductive coding found in Miles, Huberman, & Saldana (2013). First, she open-coded each teacher interview to generate an initial set codes that were subsequently collapsed the codes into a broader set of codes and shared these with an expert in mathematics education for feedback. This resulted in a final teacher codebook that was applied to both teacher interviews.

From here, the researcher conducted a theme analysis which included exporting all excerpts coded with the same code into an excel spreadsheet and re-reading the excerpts, making note of the general theme(s) coming up in that code. From this process the teacher themes were generated and written up as one summary paragraph per theme.

[1] Teachers nominated a pair of students for each designation, "turn around" refers to students who started the school year low achieving but had improved throughout the school year.

Following the analysis of the teacher interviews, the researcher completed the same process with the student interview data which resulted in summary paragraphs for the student themes.

Since the research question asks about how the teachers and students construct relationships with mathematics together, the next step in the analysis required making connections across the two data sets. This process started with a comparison of the theme paragraphs from both sets of data and then a grouping strategy to create general themes. In some cases, this process was straight forward because both teacher and student themes already matched. For example, both the student and the teacher analyses resulted in a theme around mathematical authority, so those were grouped together as a general theme across both data sets. However, for themes that were less straight forward, the researcher would group similar ones together and then generate a heading for that theme. As she made her way through the list of teacher and student themes, she would first try to place the theme into one of the already existing headings, adjusting the title of the heading to better suit the themes included. In the cases where she was unable to reasonably connect the theme to a heading, she would generate a new heading. This was an iterative process that resulted in four general themes that cover all of the teacher and student themes where the most noteworthy findings centered upon agency and authority in the classroom.

Survey Data

The survey response data was first downloaded from the Qualtrics site and uploaded into the STATA quantitative data analysis software by another member of the research team. Then, the researcher created a table for each survey question that displays the spread of student responses by both frequency and percentage. To draw connections between the interview data and survey data, the researcher combined all agree answers into one metric and all disagree answers into another.

Findings

Both the teacher and student interviews surfaced a theme around agency and authority. The teachers want their students to work with one another to make mathematical decisions, choose methods, share ideas, and come to their own understandings around the content and take control of their learning. In an effort to encourage these practices, both teachers report trying to take a step back so that students can have genuine experiences of doing math with one another – and making mathematical decisions - without the teacher as the main authority.

> Ms. Anderson: I think that if I was constantly stepping in, it doesn't... I think that that just puts me back at this position of: 'I hold all the knowledge and I'm in charge of all of this.' And there are already enough times when I am that, and I am playing that role. And I've had some experiences this year where I blatantly did something wrong, and the kids didn't say anything to me. And I was like, 'Guys, what? You let me go through that whole thing.' And they were like, 'We figured you must have been right.' And I was like, 'Well, I'm flattered that you think I'm great.' Like, 'No, if you... You guys need to trust, trust yourself.' So I think that there are so many times where I already have all that power and control that if I'm gonna let a student go to the board, I don't wanna make that be a pseudo experience. I would rather have them actually be in control of it.

Mr. Lang shares his thinking around the value of students exercising mathematical authority and sharing their work with the class, especially when the class is struggling with a particular topic.

> Mr Lang: I think the more that when we work on something and there's some sort of place where we get stuck, to have a student or a student group go and present their solutions or how they're thinking about it. We're not ready for the more— I think I read this is largely a common thread in Chinese mathematics classes where they actually look for people to do good mistakes, if you will, to present it on the board and have that be learning— I don't think we're there yet, we're probably more showing the thinking and the steps towards what's gonna be a productive solution, but to have more students talk both so that they can get their ideas heard and be seen as students that can

> bring good mathematical knowledge to the class, as well as their articulation of what they understand about math, I believe, really helps them sort of solidify whatever understanding they develop.

Although Mr. Lang would like for his students to have agency and authority in approaching math through their own different ways of thinking, he sees his class as ready to discuss correct or "productive" thinking only. Herein lies a tension, there is both a sense of freedom and confinement of student thinking embedded in his statements: he wants them to feel the freedom to express their ideas but confines which ideas he values. This focus on correct thinking and answers is reflected in the student responses in terms of their beliefs in their own mathematical agency and authority in the classroom.

The students' feelings of agency in mathematical problem solving are limited to choosing and utilizing different resources (such as calculators or peers) rather than choosing or creating their own mathematical ideas. For example, Kim, a high achieving student, explains that she is aware that she can utilize a variety of resources to succeed in math.

> Kim: I think what we need to do to be successful in our class is to ask more questions and ask for help, and really use the kind of resources we have around us, like our teachers and our peers, to help us.

This response is similarly reflected in the survey results where 89.1% of students agreed with the statement: "I am in charge of my own learning journey in math." However, only 66% agreed that "In math class I feel creativity is valued". While many students feel that they are in charge of their "learning journey", fewer see creativity valued in the classroom. If we consider creativity as original thinking, then we can understand this result to communicate that students do feel agency over their math learning more generally, but not necessarily mathematical agency, meaning, students are not expressing and utilizing their own mathematical ideas. Students feel free to utilize resources (including the teacher and peers) but when it comes to sharing their unique ideas about math (creativity) they feel less freedom.

This apparent lack of expressions of mathematical thinking is closely intertwined with the students' perception of mathematical authority. For these students, the act of deciding mathematical legitimacy was whittled down to merely deciding which answers are right and which ones are wrong rather than an interrogation of another's mathematical reasoning or justification. For example, Silvia, a "turn around" student, was asked how she when the mathematical work she is doing is right and gave the following explanation:

> Silvia: We don't know. [chuckle] I mean, I don't know when I'm doing my work, I don't know if it's right or wrong. I would probably just ask a teammate or Ms. Anderson but yeah.

Of the twelve students interviewed, eleven students believed that their peers can help them decide what is right and wrong but that their teacher is the ultimate authority on the material. Perhaps the greatest detailed expression of the teacher's mathematical authority came from Lucy's explanation of what she does to decide if her work is right.

> Lucy: Oh, I just ask the teacher time and time again like, "Is this right?" And then when the teacher sees the problem on the work we try to show, then the teacher just sits down with us and then explains it deeply and deeply like, "What you need, what's this thing called and what's that thing called?" And then you answer it and then you... And then once the teacher is like, "That's correct," then you write it down with it. You write down step-by-step, how do you do that and how you do that, which is really helpful.

The belief that the teacher is the main source of mathematical authority appears to also be shared by a portion of the students surveyed. The results showed that 42% of students agree with the statement: “The teacher is the only one that knows if I understand or not”.

The focus on correct or incorrect answers rather than mathematical thinking seemed to manifest in a fear of mistakes and struggle. The students interviewed talked about struggle as something that is negative and should be avoided. For students, struggling is a sign that you don’t understand rather than a key part of the learning process.

> Kim: Whenever I get stuck, it makes me feel frustrated, and it's really uncomfortable because I feel like I could do it, but it just stalls, and my brain is like blank.

For Kim, and other students, getting stuck brings forth feelings of frustration and an inability to keep moving forward. Silvia expressed how continued instances of struggle begin to discourage her from mathematics learning.

> Silvia: I wouldn’t say I hate math, but it is frustrating. I was raised to be always a good kid, so I always like to be really good at what I’m doing. But when it comes to math, when I don’t get something, it just feels so frustrating. I'm just like, ‘you know what? Nevermind. Forget it. I’m not doing this,’ and I kinda just get stuck with that mindset…Yeah, it is really frustrating ‘cause you’re trying to actually be present in the group and trying to help other people. But then you’re just like, ‘Welp, I’m stuck.’ And it’s kind of (pause) ugly to have to be asking other people constantly about what's going on.

This communicates a low level of student agency as the students become debilitated by their own signs of struggle. These sentiments are reflected in the survey responses, where just over half (52%) of the students surveyed responded that they agree with the statement: “It is important not to make mistakes in math”, and 77% agreed with the statement: “I feel discouraged when I get a low grade in math”. The majority of students see mistakes and struggle as negative indications of one’s math learning and math ability. Furthermore, feelings of agency are thwarted as students feel discouraged by their struggles.

Overall, both the teachers and students varied in their reports about agency and authority in the classroom. The teachers want students to have agency and authority in their classrooms and they attempt to cultivate these dispositions by asking students to present their work to the class or share their thinking with peers. However, Mr. Lang notes that he feels his class is not ready to share incorrect thinking with one another. The students explain that they do share their work with one another, but with a focus on obtaining the right answers rather than sharing their thinking. Both teachers emphasize a desire for students to see themselves and one another as mathematical authorities rather than relying on the teacher without question. However, for eleven of the twelve students, the ultimate authority lies with the teacher.

Discussion and Conclusion

Together, the teachers and students in this study have constructed particular relationships with mathematics that can be thought of in terms of identity, knowledge and practice. In terms of identity, students seem to expect to take on a role that is not entirely passive but not quite active either. They report interacting with their peers but mostly for the purposes of verifying they are getting right answers instead of for the purpose of sharing their mathematical thinking. Mathematical knowledge has taken the form of the ability to decide what answers are right or wrong. In terms of practice, teachers and students value producing right answers above exploring the value of mistakes and struggle. Teachers report focusing student attention on what’s deemed “productive”. For the students, even though they see effort as important for success, there is a tension here where students actually wish to avoid struggling and spending too much time on any one topic as this feeling of struggle becomes debilitating for them, as noted by Kim who finds that when she gets stuck, “my brain is like blank”. The lack of value placed on struggle and mistakes works against the feelings of agency for students.

The teachers feel that students are not ready to discuss mistakes and instead focus on student solutions that are “productive”. This begs the question of who are the students that are producing what Mr. Lang describes as “productive” solutions and “good mathematical knowledge”. The avoidance of engaging with struggle and mistakes keeps the focus on being right and therefore devalues asking too many questions or asking for too much support. Furthermore, it increases the risk of sharing your thinking if you are not sure it is correct. Although the teachers express a desire for students to experience agency and authority in the classroom, it appears that the development of these dispositions are thwarted by a fear of mistakes and struggle on behalf of both the teachers and the students. Students are focused on asking questions to obtain the right answers or making sure they are doing the right steps rather than engaging in collaborative mathematical sensemaking.

An expansion of what is mathematically valuable would allow a wider range of students to see themselves (and be seen) as productive and important contributors to the mathematics community and expand beliefs about who is good at math.

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