CHARACTERIZING COHERENCE WITHIN ENACTED MATHEMATICS LESSONS

Jaepil Han
University of Missouri
jaepilhan@mail.missouri.edu

Meghan Riling
Boston University
mriling@bu.edu

Hector I. Nieves
Boston University
hnieves@bu.edu

Leslie Dietiker
Boston University
dietiker@bu.edu

Rashmi Singh
Boston University
rsingh9@bu.edu

Curricular coherence has been emphasized by leaders in mathematics education as it enhances deeper understanding by enabling students to see connections between mathematical ideas. Although there are different forms of curricular coherence, the coherence of lesson has received considerably less attention. Little is known about what constitutes coherent lessons or how to measure the degree of coherence. Using the data from a larger study in which lessons are intentionally designed for coherence, we propose a tool for examining lesson coherence and describe characteristics of the lessons with different levels of coherence.

Keywords: Curriculum, Curriculum Enactment, Instructional Activities and Practices

What explains poor U.S.A. performance in mathematics according to international comparison studies? According to Stigler and Hiebert (1999), who compared enacted lessons from seven countries including the U.S.A., one of the factors that distinguishes mathematics lessons in the U.S.A. from high-performing countries is their degree of coherence. They explain:

> Imagine the lesson as a story. Well-formed stories consist of a sequence of events that fit together to reach the final conclusion. Ill-formed stories are scattered sets of events that don't seem to connect. As readers know, well-formed stories are easier to comprehend than ill-formed stories. And well-formed stories are like coherent lessons. They offer the students greater opportunities to make sense of what is going on. (p. 61)

Much of the prior research has focused more on *curricular coherence*, which refers to how mathematics topics are connected across grade levels (Schmidt, Wang, & McKnight, 2005). Coherence *within* a lesson (what we refer to as *lesson coherence*) has received considerably less attention. Our goal is to learn what constitutes a coherent lesson and to what extent one lesson's coherence differs from that of another. Using lesson data from a larger study in which lessons were intentionally designed for coherence, the present study aims to answer the following questions: When enacted lessons are analyzed for how mathematical ideas build within each lesson and how its parts are interconnected, to what extent are the lessons distinguishable? What are the characteristics of lessons in each type of lesson coherence?

Theoretical Framework

As Stigler and Hiebert (1999) suggest, mathematics curricula (i.e., lessons, units, entire courses, and so on) can be thought of as *mathematical stories* (Dietiker, 2015). Mathematical stories foreground how the mathematical content unfolds across a lesson, connecting a beginning with an ending. The sequential parts of a mathematical story form its *acts* during which students' understanding of *mathematical characters* (e.g., numbers or geometric objects), *mathematical actions* (e.g., procedures), and/or *mathematical settings* (e.g., representations) changes.

A story's coherence is the extent to which parts of stories fit with one another and come together as a whole (Richman, Dietiker, & Riling, 2019). Incoherent mathematical stories make it harder for students to see connections between lesson parts (i.e., acts) and prevent students from fully

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

comprehending a mathematics lesson. However, it does not necessarily follow that more coherent lessons are always better; students may feel boredom during a predictable lesson.

Methods

In order to learn about types of lesson coherence, we analyzed recordings of secondary mathematics lessons expected to represent a range of coherence. The lessons were taught by six teachers from three high schools in Northeastern USA. About half of the lessons were teachers' typical lessons and the rest were designed as mathematical stories, a process that we predicted would increase coherence. Data includes video- and audio-recordings of full lessons. At the end of each lesson, consenting students completed a survey describing their experience. In order to achieve maximal variation in coherence, we identified each teacher's lessons with the most positive and negative student aesthetic reactions. We also included a lesson for which the teacher had participated in analyzing a previous enactment as a story, which we thought might result in a unique form of coherence.

Members of the research team coded independently and met for consensus throughout the coding process. The team first identified *acts* by noting changes in mathematical characters, actions, or settings (e.g., when students shift to a new task). Within each act, the team identified questions that arose. For each question, researchers marked changes in what was revealed publicly, such as when a teacher asks clarifying a question when or students shift to a new task.

In order to examine the connectivity of each lesson, we created a *coherence map* using graph theory. Nodes represent acts and edges reflect that two acts contain progress on the same question(s). Lessons were then grouped based on the connectedness of their graphs. For each level, at least two research team members examined the transcripts of the lessons to demonstrate the characteristics of each level.

Findings

We identified three levels of coherence within our data set: incoherence, partial coherence, and strong coherence. Here, we present coherence maps of lessons selected to represent each coherence level and articulate features and characteristics of each level.

Level 1: Incoherence

Two lessons analyzed contain discontinuities between topics and a lack of overarching themes across tasks. One of the lessons (see Figure 1) begins with a warm-up in which students describe properties of an operation (i.e., a&b = 3a – b). Yet, these features are not relevant to the next task, about the range of a function of x. After that, the lesson has another disconnect when the focus in Act 7 shifts to an unrelated topic (percent) without explanation. Although all tasks in Acts 7-12 are about percentages, they jump from calculating percentages of numbers to calculating prices as percent discounts, and so on. No work that students do to complete one task supports them to complete the rest. It is unlikely students will connect these tasks beyond recognizing that each is about percentages. Because these tasks are so independent, there exists no obvious sequence that would support students in building an understanding of percentages.

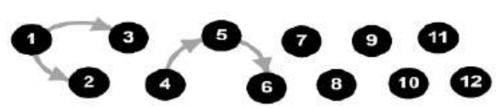

Figure 1: A Visual Model of an Incoherent Lesson

Level 2: Partial Coherence

Two of the analyzed lessons show some extent of coherence within substantial portions of the lessons. The coherence maps of such lessons contain sections with some internal connections, but no connections across sections. One of these lessons (see Figure 2) beings with students observing four graphs of systems of linear inequalities (Acts 1-6). During this activity, students work on questions like, *why are the different parts shaded?* After that, however, this question is not pursued by the teacher or students as the students work on a worksheet with other types of inequalities (e.g., one variable inequality) in Acts 8-14. In Act 15, the teacher briefly review the answers to the worksheet problems and ends the lesson. The lessons in this level include more connections between acts than the incoherent lessons do. However, these connections rely heavily on a couple of acts (e.g., Acts 6 and 15), which build some extent of coherence but not a strong amount.

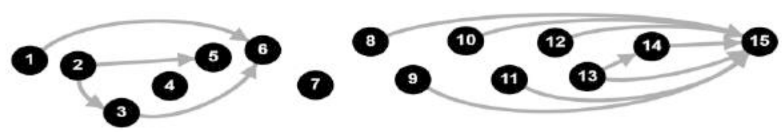

Figure 2: A Visual Model of a Partially Coherent Lesson

Level 3: Strong Coherence

Some lessons in our data set were strongly coherent. That is, a student would likely understand why they were engaged in a given activity and know how parts of the lesson connected to one another. Three sub-types of strong coherence—retroactive coherence, coherence with brief diversions, and strong coherence—are described below.

Retroactive coherence. In some highly coherent lessons, there are portions of the lesson that appear disconnected, but are later shown to be connected. These lessons are similar to partially coherent lessons in which teachers review answers at the end of the lesson, but retroactive coherence is richer because students have opportunities to make conceptual connections across ideas from the lesson. Consider a lesson with this type of coherence about repeated roots of polynomials (see Figure 3).

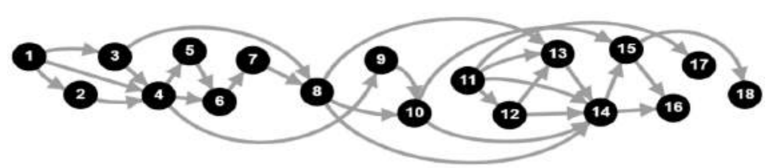

Figure 3: A Visual Model of a Retroactively Coherent Lesson

In Acts 1 through 10, students are shown a graph of a polynomial with a repeated root and work collaboratively to find its equation. In Act 11, the teacher explains that they will no longer be making progress on questions regarding the graph and distributes a new worksheet with equations to graph. In the final acts, the teacher enables students to see connections between the two seemingly distinct portions of the lesson by turning their attention to broad concepts that apply to both. Several early questions that did not refer to specific equations or graphs, but do broadly apply to them, become relevant again in Act 14. Retroactive coherence is possible because the teacher does not disclose many questions from the first part of the lesson before beginning the second part, so students may still wonder about them later on.

Coherence with brief diversions. Sometimes, most acts were connected to each other, with a few brief diversions consisting of acts that were somewhat, or not at all, connected. We found four such lessons. Two have brief diversions that appear as tails at the beginning of a lesson, typically due to teachers reviewing prior work. The others have tails later in a lesson, when teachers introduced alternate solution strategies that are not addressed as the lesson continues. An example of this second type is a lesson about exponential equations (Figure 4). In Act 15, the teacher introduces a new, efficient way of solving for x in the equation $1.04^x = 2$. The teacher asks students to think of way to find x that would be more efficient than the predominant solution strategy (i.e., guessing and checking) used in the lesson. He then solves the problem using the new method (i.e., taking the logarithm of both sides and using the power property to solve for x).

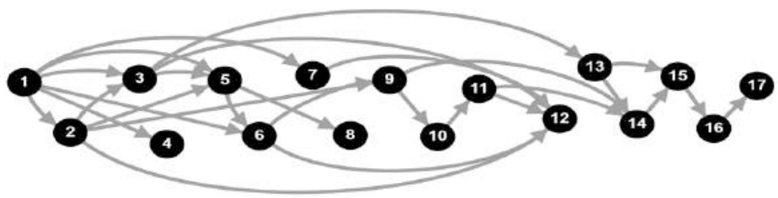

Figure 4: A Visual Model of Coherence Lesson with Brief Diversions

Strong coherence. Several lessons designed as mathematical stories displayed an incredibly high level of connection across acts with no diversions or temporary coherence gaps. The coherence map of one lesson with strong coherence is represented in Figure 5. In this lesson, students built understanding of the Rational Root Theorem by investigating potential roots. The lesson's high degree of coherence is evident in the multiple complete subgraphs (e.g., Acts 5-7). The strong coherence is due to both a set of questions from Act 1 and new questions introduced in subsequent acts that remain open for most of the lesson. The teacher permits the students to gradually explore and refine their ideas as they consider new challenges, prompting them to explore their initial questions during each subsequent task. The progressively complex nature of each new task (e.g., checking provided roots versus selecting their own potential roots) likely makes it so that students do not grow bored of their investigation.

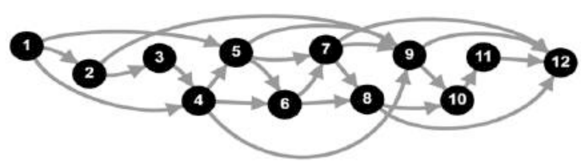

Figure 5: A Visual Model of Strong Coherence

Discussion

We do not claim that these three types of coherence are discrete. There might be additional intermediate levels, or even a continuous coherence spectrum. The presented levels are only samples of this possible spectrum; our goal is to present a way to describe the coherence of a mathematics lesson. Lesson coherence is not only a quality indicator of a mathematics curriculum but also a useful dimension for making a lesson more captivating in terms of student engagement. Increasing coherence requires purposeful design and management of mathematical inquiry. Through coherence mapping, lesson coherence that is often implicit can be visualized so that teachers will be able to see how they can make stronger connections between parts of a lesson.

Acknowledgments

This project is funded by the National Science Foundation, grant #1652513. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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