USING STRIP DIAGRAMS TO SUPPORT PROSPECTIVE MIDDLE SCHOOL TEACHERS' EXPLANATIONS FOR FRACTION MULTIPLICATION

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In this report, I describe how prospective middle school teachers created strip diagrams to solve fraction multiplication problems. I analyzed classroom videos from a year-long content course in order to determine what how teachers drew the diagrams and found four critical features of the drawings. I explore how they used the features as they drew and explained their thinking.

Keywords: Teacher Education–Preservice, Rational Numbers, Representations and Visualization

In North America, representations are a critical component in school mathematics (La Secretaría de Educación Pública, 2011; Ontario Ministry of Education, 2005; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Researchers have also emphasized the importance of representations in mathematical thinking (Cuoco, 2001; Janvier, 1987). Both teachers and students use representations to help them solve and make sense of problems (e.g., Lobato et al., 2014), communicate their ideas (Roth & McGinn, 1998), and participate in mathematical activity especially if their language is not the privileged language in the classroom (Turner et al., 2013) However, there are some roadblocks presented in the research on teacher knowledge about representations. Researchers have produced little evidence that teacher preparation programs (both for practicing and prospective teachers) prepare them to successfully integrate representations in the classroom (Stylianou, 2010). In this study, I provide a case demonstrating prospective teachers can sensibly engage in mathematics with representations. In particular, I ask the following questions: *How do prospective teachers draw strip diagrams to solve fraction multiplication problems in class? How do they use the strip diagrams to solve fraction multiplication problems?*

Theoretical Framework

Researchers who have studied representation use in class (Izsák, 2003; Saxe, 2012) have generally agreed to distinguish what is being represented and what is "doing" the representing (cf. von Glasersfeld, 1987). In this study, I refer to representations as observable geometric inscriptions that can be referred or pointed to as the object of discussion (Goldin, 2002). It is this indexical and communicative nature of representations allowing students to explain their thinking and for others to engage in another's way of reasoning. When students create a display to represent their thinking, they also communicate with them. In other words, they tailor their display with an audience in mind (Saxe, 2012) and thus students select salient features to highlight and point when creating and talking with representations. Additionally, I frame representations as culturally and historically rooted. A representation's cultural and historical meaning stems from how communities interact with an inscription over time (Blumer, 1986). For example, a class can ascribe the meaning to the inscription "=" as "execute the arithmetic to the left" if they are continually asked to solve result-unknown problems over time.

Data and Analysis

I analyzed four weeks of instruction from a sequence of two mathematics content courses for prospective middle school teachers (PSMTs) enrolled in a teacher education program. The same instructor taught both courses. The objective of the course was to strengthen the students'

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mathematical understanding of middle school topics. The 13 PSMTs enrolled in the course were predominantly white women. The students were expected to use a multiplicand-multiplier definition for multiplication, notated by equation $N \cdot M = P$ (Beckmann & Izsák, 2015). In this equation, N denotes the number of base units in one group (the multiplicand), M denotes the number of groups (the multiplier), and P denotes the total number of base units in M groups. The class also defined the fraction a/b as the quantity formed by a parts of size 1/b. They were also expected to explain with drawings rather than memorized algorithms or symbol pushing.

The main data corpus for this study was video and audio-recorded lessons from class. The primary analytical techniques I used were modified from Saxe et al., (2015) and focused on identifying forms and functions of the representations. In this report, I focus on how "coarse forms" were drawn. A coarse form is a set of inscriptions used sequentially. When listening to explanations during discussions, I segmented the drawings based on how the PSMT described the sequence of drawings as indicated by utterances such as "I did this…and then drew this…" (Fig. 1). I then found coarse forms that were similar across all the drawings.

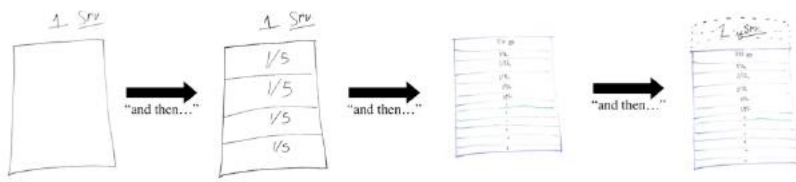
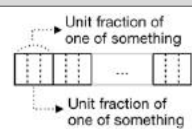
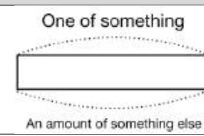
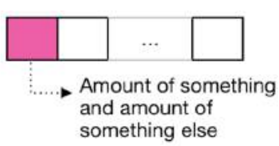
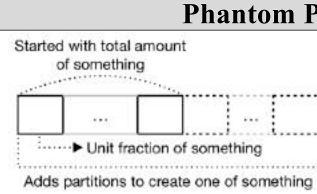


Figure 1: An example of distilling coarse forms

Results

I summarize the four main coarse forms I characterized in Table 1. I then describe how they were used for multiplication problems and illustrating it with student work.

Table 1: Coarse forms characterizing strip diagrams for fraction multiplication

Form	Partitioned Parts	Dual Function of a Strip
Schematic		
Description	Equi-partitioned rectangle with each part partitioned further	A strip, rectangle
Function(s)	Create a particular number of parts	Represent two different quantities where the amount of one quantity is one
	Dual Function of a Part	**Phantom Parts**
Schematic		
Description	One part of an equi-partitioned rectangle	Equi-partitioned rectangle then more parts are added
Function(s)	Part represents an amount of a quantity and another amount of another quantity	Determine the size of a part or partitioned part

Explicitly describing two quantities. As PSMTs drew strip diagrams, they described both the full strip and parts with respect to two quantities, the multiplier and multiplicand. Analytically, I found the Dual Function of a Strip and a Part present in all the strip diagrams. At the beginning of the sequence of lessons, the instructor formally introduced the multiplier-multiplicand definition of multiplication. Throughout the period, the instructor constituted the norm of identifying the group and units in the PSMTs diagrams.

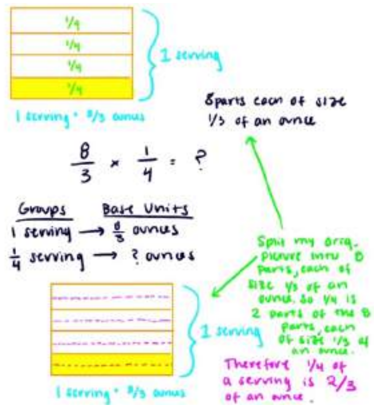


Figure 2: Hannah's diagram for the Bat Milk Cheese problem

Hannah demonstrated working with two different quantities while solving the Halloween-themed problem, "You had 1/4 of a serving of bat milk cheese. One serving of bat milk cheese is 8/3 ounces. How many ounces of cheese do you have?" She began her drawing by showing one whole serving or one group and showing the size of the group. She wrote out the definition "8 parts each size 1/3 of an ounce." She then realized she wanted to show eight parts in the strip and noticed she already had four parts. She partitioned each part into two smaller parts to show eight parts. She ended by saying there are two-thirds ounces in one-fourth of a part because there are two parts, each one-third of an ounce in the yellow part indicating one-fourth of a serving.

Determining the number of parts needed. The PSMTs wrestled with the appropriate number of parts required to solve the problems. They thought through the number of parts they created from the multiplicand when it was not divisible by the number of parts they needed.

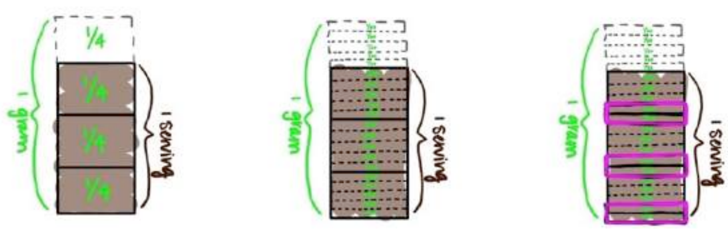


Figure 3: Elizabeth and Jack's diagram for the Blank Multiplication problem

For instance, Elizabeth and Jack were thinking about the number of parts while working on a multiplication problem, "One serving of ___ is 3/4 ___. You had 2/5 of a serving. How many ___ of ____ did you have?" In this blank problem, the PSMTs were invited to provide their own quantities. During small group discussion, Elizabeth explained that they started with a strip representing one gram partitioned into four and shaded three parts representing three-fourths of a gram or one serving. She wanted to find two-fifths of three-fourths. She partitioned each fourth part five parts then Jack suggested she should "get" two partitioned parts from each parts to get two-fifths of the serving, she highlighted two of the three one-fourth serving partitions. Partitioning of the parts was prevalent in almost all the diagrams created.

Determining the size of a part. The last form, Phantom Parts, emerged towards the end of the sequence of lessons. PSMTs began with a strip representing one group and representing an amount of base units as indicated by the multiplicand. When the multiplier was less than one, the PSMTs they needed to add "Phantom" parts in order to determine the size of the parts. They drew out additional parts to describe the product with respect to the base unit, thus they had to draw a whole base unit to describe the size of the product in terms of base units.

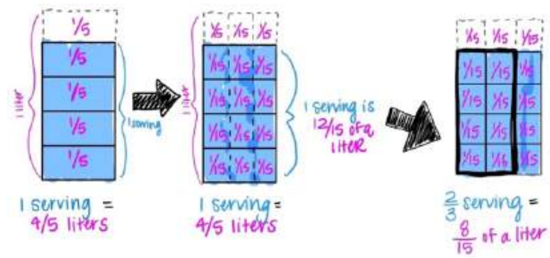


Figure 4: Elizabeth's diagram for the Goblin Goo problem

Consider Elizabeth working on the problem "You had 2/3 of a serving of goblin goo. One serving of goblin goo is 4/5 liters. How many liters of goblin goo do you have?" First, she drew a strip representing both one serving and four-fifths of a liter, similar to Hannah's use of the Dual Function of a Strip in the previous problem. She highlighted four parts to show four-fifths of a liter or one serving as seen in Figure 3. Upon partitioning each fifth into three, she labelled and described each partitioned part as one-fifteenth of a liter. She finally highlighted two-thirds of a serving by highlighting two columns of the partitioned serving to show two-thirds of a serving as eight-fifteenths. While she did not express any reason for changing her diagram from Phantom part to incorporating the Phantom part in the initial strip, the next day while talking to one of the graduate students about this problem, she said, "I think it helps understand how many parts there are of a liter. 'Coz that's why it was confusing to me was putting in in twelfths because that's not twelfths of a liter… You can do much less work if you just understand that there's a pretend liter… just go with liters the whole time. Don't change your wholes last minute."

Discussion and Conclusion

The results of this study provide a characterization of how representations evolve over time. In this case, the forms of strip diagrams evolved. The PSMTs' explanations for multiplication were rooted in two practices—using strip diagrams and a definition of multiplication. Strip diagrams evolved over time to address certain features of both the problem and diagram. By using a quantitative definition for multiplication, they were able to describe parts of the diagram (strips and partitions) with respect to two quantities. Some future steps for both researchers and teachers can be drawn from this report. When analyzing inscriptions, researchers must attend and be explicit about the grain size of the inscription. I have shown how describing coarse forms enabled me to describe continuities and discontinuities between points in time in order to characterize how representations change and potentially teaching opportunities for new forms and functions to emerge. However, although this was helpful for me analytically, such an analysis emerged from the data I had i.e., how these particular PSMTs talked.

Acknowledgments

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