# MOVING TOWARDS MEANING MAKING IN MULTIPLICATION: A PRELIMINARY REPORT OF AN INTERVENTION IN NUMBER SENSE 

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Rejecting pedagogies of deficit built around deficit mythologies about the mathematical potential of students with learning disabilities (Lambert 2018), this study will document an intervention designed to engage students in mathematical problem-solving and discussion, building student computational skills as well as number sense and participation in the mathematical practices. In this paper, we provide close analysis of the development of a fourth-grade student who demonstrated growth in participation and conceptual understanding of multiplication across the intervention.

Keywords: Equity and Diversity; Special Education; Number Concepts and Operations; Instructional Activities and Practices.

Research on the mathematical learning of students with disabilities has focused on medical deficits within children and prescribed behavioral mathematics teaching to remediate these deficits (Lambert \& Tan, 2020). Locating the problem not within individual students, but within limited access to opportunities to make mathematical meaning, we designed an intervention for students with disabilities ages 8 through 11 significantly underperforming in multiplication and division and including students with disabilities. This intervention is designed to engage students in mathematical problem-solving and discussion, building both student number sense and participation in the mathematical practices. Our full study explores the growth of student strategic thinking, accuracy for multiplication and division, and participation in mathematical discourse. In this brief report, we focus on one student whose measurable math score did not increase during and after the intervention. While she displayed no measurable growth in her math score, she displayed growth in participation and strategy development. Our research question for this brief report focused on one student: How did one student participate in a mathematics intervention designed to promote student meaningmaking and discussion? What shifts in participation are in evidence? What shifts in conceptual and procedural understanding of multiplication?
Schools in the US are being asked to provide intervention within Multi-Tiered System of Support (MTSS) in mathematics. However, interventions are often not aligned with classroom instruction based on Common Core State Standards, creating difficulties for students who must make sense of different approaches to mathematics. Our intervention using number strings is designed to align with a focus on meaning making in the curriculum. The intervention consisted of 8 sessions of number strings (Lambert, Imm \& Williams, 2107) designed and facilitated by undergraduate tutors after 6 hours of professional development led by the first author. Each tutor was observed 2-4 times by a member of the research team and offered feedback. In addition, all tutors participated in a session in which they analyzed the participation of the students in their small group. Future analysis will focus on the teaching moves of the novice tutors.
A number string is a short ( $15-20$-minute) daily instructional routine in which a teacher presents a carefully designed sequence of problems one at a time for children to solve mentally (Lambert, Imm \& Williams, 2017). Instead of interventions that focus on direct instruction, number strings provide opportunities for students to engage in mathematical discourse, both in describing their own strategies and connecting with the mathematical strategies of others. Research on number strings has found that students participating in number string routines are able to adopt new strategies (O'Loughlin 2007) and make connections between conceptual understanding and procedures. Studies

[^0]on number strings have not previously focused on students with disabilities or students who are significantly underperforming in mathematics.
One issue in the assessment of students with disabilities is the difficulty of capturing growth that may be unusual in its learning trajectory or slower than peers. Van Geert and van Dijk (2012) describes the importance of moving beyond group-level data in understanding the variability for students that may exist at the level of strategies and engagement, recommending collecting data on inter-individual variability to better understand strategic change. We document conceptual growth through analysis of student participation and discourse, including attention to non-verbal communication. We used aspects of the coding scheme by Ing at al. (2015). A Complete Share was an answer that was accurate and explained in enough detail that we could confidently code the strategy. A Partial Share was either inaccurate or did not include enough detail that researchers could determine the exact strategy path of the student. We added the last two categories to track students who had nonverbal engagement in the problem. Nonverbal captured moments in which we could see evidence of nonverbal engagement, yet students did not verbally share in discussion (such as students counting on fingers). No Engagement was coded if the student did not demonstrate verbal or nonverbal engagement.

## Methods

The study was situated in grades $3-5$ at an elementary school in California. Demographics are as follows: $76.3 \%$ are Socioeconomically Disadvantaged, $14.4 \%$ are Students with Disabilities, 58.9\% are English Learners, and $9.9 \%$ of students are Homeless. The majority of students at the school are Hispanic (88.3\%) with the second largest demographic category being White students (7.5\%). The full study included 12 student participants in 3rd grade, 6 students in 4th grade, and 18 students in 5th grade. 12 students had current IEPs, with an addition 4 students in the referral process. Each group met for 8 sessions, twice a week for 4 weeks. We collected two primary kinds of data: a) researcher-created multiplication and division paper and pencil assessments (Multiplication + Division CCSS CBM Math Assessment) with all students taking the assessment three times (pre, during and post intervention), and b) video records of the tutors teaching the number strings to document student participation and strategy development. In order to ascertain growth in student accuracy, we scored the MD-CBM before, during and after the intervention. After analysis of the first MD-CBM assessment, the first researcher met with the classroom teachers to decide the students who would be placed into the Tier 2 intervention. We assessed student use of strategies and participation in mathematical problem-solving and discussion through analysis of transcripts. Two authors each coded the small group we present in this paper, resolving any discrepancies. We will determine intercoder reliability for the final paper.

## Findings

This paper is a case study that focuses on one student (Inez) within one small group of 6 students in a fourth-grade class taught by undergraduate tutor Yola (all names are pseudonyms). Comprised of students with and without disabilities, the students in this small group had the lowest scores on multiplication in their class. Inez is Latina and classified as an English Learner, as well as a student of significant concern for her classroom teacher. The teacher noted that Inez rarely shared in math class and seemed to have significant issues with number sense. Inez appeared eager to participate in this small group, even when she did not share. She seemed particularly to enjoy talking to Yola.
In the first two sessions, Inez did not volunteer to answer questions. She shared twice when called on. Unlike her peers, she did not use her fingers to keep track as she skip counted. Instead, we could see her subvocalizing her counting and losing track. In discussions, we wondered if Inez needed support to help her keep track of her count. Starting in the second session, Yola passed out card stock
arrays for the students. Inez began using these arrays to keep track of her counting. However, Inez seemed to need additional experience with the arrays. Initially, Inez counted each square as 2. For much of the first 6 sessions, Inez demonstrated her ability to count by 2 s and 5 s , but not other numbers. Inez appeared to prefer counting by 2 s and 5 s so much that she used this strategy to solve problems that neither 2 or 5 were factors. Asked to solve $9 \times 5$, she got the answer of 10 by skip counting by 2 s , because "I thought it would be easier to count by twos." In discussions with Yola, we decided at the end of Session 2 that Inez needed to sit closer to Yola, who supported her in using her fingers or the array as a tool. Yola also spoke to Inez during turn and talks, which seemed to support Inez sharing in the small group. This shift seemed to mark a pronounced difference in engagement (Table 1) from a lack of engagement in the first two problems to a more sustained engagement in mathematical discussion in the subsequent sessions.

# Table 1: Shifts in Inez's Participation in Mathematical Discussion 

|  | Session 1 | Session 2 | Session 3 | Session 4 | Session 5 | Session 6 | Session 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Complete Share | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| Partial Share | 1 | 1 | 4 | 4 | 6 | 5 | 3 |
| Nonverbal | 0 | 3 | 0 | 2 | 0 | 0 | 1 |
| No Engagement | 5 | 1 | 1 | 0 | 0 | 0 | 0 |

Through close analysis of Inez's strategies across the 7 sessions (one session was not video recorded), we saw evidence that Inez developed her understanding of arrays. While in the beginning she did not count squares by ones successfully (by counting boxes as 2 s or 5 s ), she was able to do so by Session 5. She also developed an understanding of the connection between skip counting and multiplication by groups. Twice, Yola represented Inez's skip counting numerically and connected that to the representation of the array. The first time Yola did so, Inez stopped, stared at the array and the skip counting represented next to it, and said, "What the heck?" The next session, Inez again counted an array by 2 s , and then miscounted, getting an answer of 62 for $6 \times 5$. Yola listened to Inez's strategy, and then remodeled it on the array keeping track of the numbers. Inez again appeared to be provoked into disequilibrium by the tutor's representation of her strategy, saying, "I went really really far." Connecting visual and numerical representations of her own strategy appeared to make Inez's own thinking visible to her, thus allowing her to understand her own thinking as reflected by the tutor's representations.
While Yola seemed to make supportive moves to increase Inez's participation, as well as to model her thinking to make it visible, Yola described having significant difficulty understanding and representing Inez's strategies. While there were instances in which Yola pressed for explanation, there were more instances in which Inez shared an incorrect answer and Yola did not ask her to elaborate. In further analysis, we will determine which teacher moves within the number strings routine were most challenging for novices to enact. We suspect that pressing a student for further explanation when that student has a pattern of strategies that do not make sense to the teacher might be a particularly challenging teaching move to enact.

## Discussion

Our intervention aims to increase the mathematics achievement of students with disabilities and students whose performance is significantly below grade level, but not with instructional practices that focus on memorization or procedural learning. Instead, we investigated the use of a number string to develop multiplication and division computation simultaneously with number sense. In this paper, we demonstrate how one student significantly below grade level in mathematics grew in her use of mathematical strategies and her engagement through participation in a number string routine.

Further analysis will include latent class modeling to determine growth patterns for particular subgroups of students. We also will document overall growth, using group averages as well as close qualitative analysis of strategy growth. Finally, we will analyze the teaching moves of the novice teachers to determine the effectiveness of the professional development we provided for tutors. We also plan to analyze how Inez's emergent bilingual status could better have been leveraged in her learning. Most importantly, we seek to better understand how to provide mathematics intervention for students who need more support engaging in meaningful mathematics.

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