PROSPECTIVE TEACHERS' APPROACHES TO PROBLEM-SOLVING ACTIVIES WITH THE USE OF A DYNAMIC GEOMETRY SYSTEM

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Using digital technologies when working on problem solving tasks allows students to engage in different ways to explore mathematical concepts as well as analyzing multiple approaches that emerge while solving the tasks. For this study, it was important to document the extent to which prospective teachers become aware of the potential of a Dynamic Geometry System (DGS) as a problem-solving tool. To this end, six prospective teachers participated in a series of problem-solving activities meant to be approached by using a DGS. Even simple tasks offer ample opportunities to explore mathematical concepts when representing them within a digital medium, and in turn, the DGS' affordances influence the way participants pose mathematical propositions and validate them while solving and extending problems.

Keywords: Technology, Problem-solving, Geometry, Teacher Knowledge.

Introduction

The introduction of a technological element in the mathematics classroom modifies the way concepts are addressed, thus creating a perturbation within the teaching system. Laborde (2002), mentions two aspects that need to be considered when a certain technology is to be introduced in a learning environment: The domain of knowledge (how are mathematical objects interrelated through such technology? Which aspects are preserved, and which ones are modified?) and the teacher-student interaction (What is the purpose being considered when using that technology? Is the technology used for learning or is it used to support the teacher's discourse?). Technological tools such as Dynamic Geometry System (DGS) have the potential to open up multi-interpretations of mathematical knowledge (Leung, 2017), objects within the DGS do not appear as exclusively virtual to students, but become materialized (Moreno-Armella & Hegedus, 2009) and, therefore, subject to experimentation. One of the most notable outcomes from the study of mathematical objects in a dynamic environment is the emergence of alternative ways to justify mathematical relations (i.e. using *point dragging* to verify if a figure holds a geometric property in certain conditions). Santos-Trigo (2019) mentions that the use of digital technologies in learning environments demands addressing what new pedagogies are needed to frame mathematical working in which learners participate in the construction of mathematical knowledge. To this matter, problem-solving activities can be exploited with the systematic use of digital technologies, allowing teachers and students to examine mathematical tasks from different perspectives that include a plethora of concepts, resources and representations (Santos-Trigo, Camacho-Machin & Olvera-Martinez, 2018). However, teachers' perspective on the nature of mathematical knowledge and the role of digital technologies will define the ways in which students interrelate conceptual knowledge when solving mathematical tasks. Thus, teachers need to rethink the nature of mathematical activities in classroom when students solve problems using a DGS (Moreno & Llinares, 2018). For this study, six pospective teachers participated in a series of problem-solving activities with the support of a DGS. To this effect, the research question that guided this work was: What are some of the ways prospective teachers explore mathematical ideas when solving problems using a DGS?

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Theoretical perspectives

Mediating tools are not epistemological neutral (Moreno-Armella & Sriraman, 2010). In a DGS, motion becomes a key element of mathematical representations. Therefore, a tool like a computer affects the cognition of the user, it reorganizes her ideas. In this way, the computer can no longer be considered as an agent that "does the task of the student" but provides students with a cognitive tool (Moreno-Armella & Sriraman, 2010). In a problem-solving environment, a DGS has the potential to enhance the use of heuristics like analyzing multiple particular cases, and fosters different problem-solving episodes like generate, explore and validate conjectures (Aguilar-Magallón & Poveda, 2017; Santos-Trigo & Moreno-Armella, 2016). These actions, however, are shaped by the subject's expertise in using the tool. Throughout all problem-solving activities, it becomes important to pay attention to the transit in learners' use of empirical approaches to the construction of geometric and analytic arguments to support results (Santos-Trigo, 2019). Santos-Trigo & Camacho-Machin (2013) proposed a framework to characterize ways of reasoning that emerge as result of using computational technology in problem-solving via four episodes: (a) comprehension episode, in which the solver needs to think of the task in terms of mathematical relations and how to use the DGS' affordances to represent the problem (to generate a dynamic configuration); (b) problem exploration episode, where the tool is used to obtain empirically-generated conjectures; (c) search for multiple approaches episode, where students need to think of different ways to solve a problem in order to develop conceptual understanding of mathematical ideas; (d) Integration, a reflection of the different processes involved in the previous episodes.

Participants, methods, and procedure

The purpose of the study was to examine how high-school prospective teachers use a computational tool as a means of exploring multiple concepts derived from solving mathematical problems. Thus, this study is oriented to the analysis of cognitive processes exhibited by the participants and therefore, is of a qualitative nature.

Six prospective teachers participated in a problem-solving course as part of a master's degree program at the CINVESTAV-IPN (Mexico City). They all had completed a university degree akin to mathematics and were attending the first semester of the program. The activities were conducted for 8 sessions of 3 hours each. The prospective teachers were encouraged to use a DGS (GeoGebra) as the main problem-solving resource. Firstly, they worked individually or in small groups and, subsequently, they presented their work to the group in plenary. Additionally, the participants were asked to prepare a report of their work in a text file and submit it to a google classroom platform. Data were collected through the information of the teacher's reports and the video recordings of the sessions.

In this research report, I focus on the prospective teacher's performance related to the following problem: Let ABC be a right triangle with perimeter 12. What are the lengths of the sides such that the triangle has maximum area?

It is important to note that, prior to the problem, participants worked on GeoGebra several construction problems collectively in plenary guided by the instructor. In this sense, they had knowledge about some of the DGS' affordances.

Results and discussion

To present the results of this study, the participants' work was structured around the problem-solving episodes described by the framework of Santos-Trigo & Camacho-Machin (2013). Also, it is also worth noticing the extent to which the use of a digital tool like GeoGebra modifies the way mathematical statements are stablished and validated (Santos-Trigo, Camacho-Machin & Moreno-Armella, 2016).

Comprehension episode. Most participants showed mainly algebraic procedures meant to find the function area $A(a)=\frac{36a-6a^2}{12-a}$. Subsequently, they didn't have any difficulties in finding that $A'(a)=0$ if $a=12-6\sqrt{2}\approx 3.515$, which is the measure of both legs of the right triangle with perimeter 12 such that its area is maximum. However, they were asked to represent the problem in a DGS. That is, they needed to construct a right triangle with perimeter 12 within GeoGebra. All of them showed the following procedure: define a slider a, and draw a circle with center A and radius a. Thus, the radius AB has side a. From B, trace a perpendicular line to AB. This line intersects a circumference with center B and radius $\frac{12(6-a)}{12-a}$ at point C. As a result, a right triangle ABC is obtained, such that its perimeter is always 12 (Figure 1).

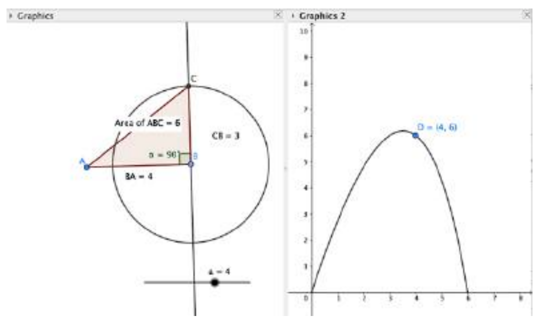

Figure 1: Dynamic configuration for Problem A

Exploration episode. How can the tool's affordances be used to find an empirical answer? Once the participants had a dynamic representation of the problem, they defined the point D: (a, t_1), where t_1 is the area of the triangle ABC. When moving the slider a, point D's trajectory can be visualized through the locus command. In consequence, the value of a can be adjusted so that (visually) point D is located at the vertex of the parabola.

Multiple approaches episode. The way this problem was been represented did not depend on the DGS' affordances, since the key aspect of the statement of the problem was approached in an algebraic way. Thus, participants were asked to re-interpret the given perimeter as a line segment of length 12 instead of a number. Using the coordinated axes, they traced AB with length 12 and placed a point C on the segment. In this way, AC is one of the sides of the triangle and by placing a point D on CB, segment AB will be partitioned into three segments. Are these segments always the three sides of a triangle with perimeter 12? It is important to see that this question does not appear when solving the problem by algebraic means, but in a digital medium, it is important to exploit the opportunities it offers to explore mathematical concepts. Figure 2 shows a triangle ACE such that $\overline{AE}=\overline{DB}$ and $\overline{EC}=\overline{DC}$. Whenever $\overline{AC}>6$ (or more generally, $2\overline{AC}>|AB|$) circumferences A and C will not intersect and, therefore, there will be no triangle. This is a crucial condition that must be stated when working with students because the motion of C must be limited in a way that ACE is always a triangle with perimeter 12 (or AB, in any case). Even though ACE is not necessarily a right triangle, it can be observed that when moving D, there is a position that results in angle AEC to be right. How to place point D such that the triangle is also a right triangle?

At this point, participants struggled to use an element of the dynamic configuration as a resource to obtain information about the mathematical relations involved. After working in small groups, two participants used the command *locus* to note that the motion of point E, because of moving D along BD, *seemed* to be an ellipse (Figure 3). What arguments could be used to support the validity of this conjecture? For different positions of *D*, it holds that $,AE.+,EC.=|CB|$, which means the sum of the distances from point *E* to *A* and *C* is always constant. Hence, point *E* moves on an ellipse with foci *A* and *C*.

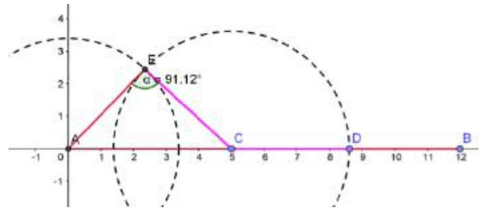

Figure 2. Drawing a triangle of a given perimeter

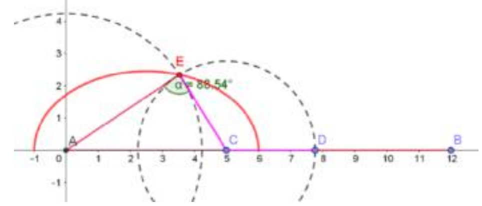

Figure 3. Locus of the point E as D moves along BC

Reflections on the problem. Two elementary forms of using loci within a DGS can be seen as crucial elements for concrete problem-solving strategies. On the one hand, the intersection of the ellipse and the circumference obtained in the analysis of the problem is a way of consolidating the heuristic of relaxing the conditions of a problem through an intersection point that unifies the solution of two subtasks (drawing a right triangle and tracing a triangle with a given perimeter). On the other hand, using a locus that represent a variation phenomenon can serve as a departure point for learners to make use of different geometric and algebraic resources to build and make sense of a robust dynamic configuration. The DGS also provides a scenario where mathematical discussions can be constantly extended. For instance, tracing the triangle considering AC as the hypothenuse rather than the side or restating the problem into considering an isosceles triangle instead of a right triangle with fixed perimeter.

Concluding remarks

In a digital medium, mathematical objects involved in the dynamic configuration of a problem become executable and react to the user’s actions. This, in turn, allows the users to further extend their reflections or to find alternative approaches to the problem. In Problem A, the use of loci was underpinning in the formulation of conjectures and participants had to validate mathematical propositions stated in terms of the DGS affordances: what arguments can be stablished such that a certain property holds for the dynamic configuration when movable points are dragged? Prospective teachers were able to explore the concept of ellipse as a resource that can be useful in tasks related to the construction of triangles with a given perimeter. What is more, the concept of locus became a way to organize the use of problem-solving strategies such as solving similar simpler problems, simplify the conditions of the statement or solving many cases. As the activities developed, they became more prone to experiment with the tool and to find different solution paths that could open up for different kinds of mathematical discussions.

Teachers need to be exposed to environments where they can experience at firsthand how a digital technology affects the organization of mathematical ideas. When students use digital artifacts like a DGS, they rely on them to bridge the gap between mathematical ideas and their personal experiences through actions that can develop in the medium such as dragging or measuring attributes. If digital classrooms are to be successful, then teachers need to be fully aware of how these processes develop and relate to the generation of mathematical knowledge (Monaghan & Trouche, 2016).

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