TOWARDS REFLECTION ON THE PRACTICE IN A LEARNING COMMUNITY OF HIGHER-LEVEL MATHEMATICS TEACHERS

HACIA LA REFLEXIÓN SOBRE LA PRÁCTICA EN UNA COMUNIDAD DE APRENDIZAJE DE PROFESORES DE MATEMÁTICAS DE NIVEL MEDIO SUPERIOR

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In this report, the authors describe the creation of a learning community for mathematics teachers with the purpose to improve their knowledge about mathematics and its teaching. It also includes a description of how these teachers have structured reflection sessions on class planning, learning activity trials, analysis of teacher performances, and how to improve the design of the teaching sequence. An important result of the first 'Teaching-Reflection Cycle' carried out within the community is that learning about pedagogical content knowledge can be promoted.

Keywords: Teacher's knowledge, reflection on practice, teaching of functions.

Shulman's influential framework (1986, 1987) on the components of effective teachers' knowledge helped stimulate research to improve teaching in many subjects. In particular, for the teaching of mathematics, there are two key investigations. Ball, Thames, & Phelps (2008) extended Shulman's categories for the teaching of mathematics and to build a framework of mathematical knowledge for teaching (MKT). Rowland, Huckstep, & Thwaites (2005) adopted another approach also based on Shulman's work, they built a framework known as The Knowledge Quartet (KQ) focusing on the teacher's mathematics knowledge that emerges within the classroom.

High school teachers frequently do not receive pre-service pedagogical training. In Mexico, teachers of this educational level, start teaching without any type of instruction. They get their pedagogical content knowledge in practice. Courses are often provided. However, the rationale of the lectures is not well planned and sometimes topics are disconnected from the curriculum that teachers follow in their schools (Sánchez & Huchim, 2015; Sosa & Ribeiro, 2014).

Researchers have sustained that mathematical knowledge for teaching could improve the teaching of mathematics (Askew et al., 1997; Rowland et al., 2000).

Considering the aforesaid situation, a group of mathematics teachers –from a Mexican high school– have decided to create a learning community with the aim to ameliorate their teaching by improving their knowledge for teaching mathematics.

For the authors, the 'Learning Community' is a group with a shared identity, characteristics, or purposes. The learning is the objective that brings members together and gives meaning to the community for different purposes (Valdés, Pilz, Rivero, Machado & Walder, 2014, p. 56). A learning community provides: (1) a framework for teachers to learn and develop knowledge together, (2) opportunities to take advantage of teachers' tacit knowledge and make it public to be shared and criticized, and (3) support to be more than technicians who implement others' ideas to become thinkers, researchers and conceptualizers (Elbaz, 1983; Schön, 1983; Wood, 2007).

The purpose of this report is to inform the process of the creation of a learning community; how teachers used reflection to design, apply, and discuss teaching activities and how evidence of teachers gaining knowledge emerges during this process.

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Mathematics Teachers' Learning Community

A group of 12 mathematics teachers from a high school level, who works on the same educational center, participated in an academic call to improve their employment status. Aware that they lack the necessary knowledge for teaching, those teachers created a learning community as a means to increase their knowledge and to obtain a promotion. However, during the first period of activities carried out in this community, only 5 teachers continue working together.

To structure the community, teachers used the method employed in the Keli Lesson Study, part of China's professional development program Xingdong Jiaoyu (Huang & Shimizu, 2016, p. 395). The program consists of two stages: practice and reflection. Like the Chinese approach, the community members also focus on professional development through reflective processes that center on how mathematical knowledge manifests itself in practice, and thus teachers gradually transform their practice. An important aspect of the learning community is that there is no institutional intervention, as it is a self-managed initiative of the teachers.

Schön (1983) differentiates two types of reflection that can occur and determine professional knowledge of an individual: reflection in action and reflection on action. Reflection in action is the process of monitoring and adapting behavior in context, while reflection on action is the process of evaluating what has already been done.

Community in Action

The structure of the activities carried out within the community that will be referred to as 'Teaching-Reflection Cycle' is composed of 4 stages:

1. A teacher designs or proposes a Hypothetical Learning Trajectory (HLT) (Simon, 1995) to be revisited by the other members and to plan its trial in the classroom.
2. A member of the community tries out the HLT, and another colleague video-records it.
3. Community members analyze the video and reflect on the teaching activity carried out in the classroom by their colleagues. To think about the teaching process, the teachers use a protocol structured by the researcher (who is a member of the community). Teachers also make suggestions to modify the HLT to apply it later on. All the community sessions are recorded on video and the totality constitutes research data.
4. A process of individual insight reflection is performed.

Activities carried out in a Teaching-Reflection Cycle related to a minimum area problem

1. During the first meeting for the aforementioned cycle, teachers agreed to apply an HLT based on the quadrilateral area problem (see Figure 1) proposed by Lola, a member of the community. She knew the problem from a master's course she took and has implemented it in her classroom several times. The members of the community agreed to deal with it in Mario and Tadeo's groups, but they switch activities. Mario taught the HLT and Tadeo recorded the session in the video.
2. In a second meeting, members of the community watched that video, analyzed and discussed Mario's intervention, and reflected on the teaching and learning processes carried out. They also gave Tadeo's ideas for the next trial.
3. Tadeo taught the HLT in Mario's classroom and Mario video-recorded the trial.
4. In the course of the third meeting, teachers watched the second video, discussed and reflected on both interventions, and reached conclusions concerning how other teachers can use the HLT in their classrooms.
5. Both teachers made an individual insight reflection about what the whole process contributed to their own teaching practices.

Stages 1) and 2) retrieving Schön (1983) are the moments of reflection in action in which teachers analyze and discuss the relevance of the HLT and its implementation. Stages 3) and 4) is the moment of reflection on action, in which teachers evaluate what has been done to make necessary changes considered for future applications.

The Minimum Quadrilateral Area Problem

The HLT of the quadrilateral area problem, included in Figure 1, was designed with the next characteristics:

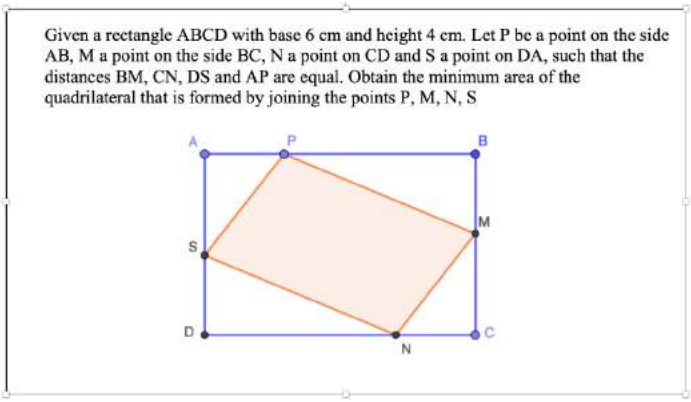

Figure 1: Problem: Compute the minimum area of a quadrilateral

1. Learning goal: Solve the geometry problem by proposing a quadratic function model. In the solving process, students will observe tabular, graphic, and algebraic representations of the quadratic function and learn how to find its minimum.

Learning activities will be applied in a two-hour session as follows: (1) Translate word problem to geometric language, (2) Area computation, (3) Tabulating task, (4) Graphing, (5) Generalization, (6) Identification of the minimum area value.

2. The hypothesis of the learning process: At the end of the activities, students should learn three representations of the quadratic function that models the problem: tabular, graphical, and algebraic, and find the minimum area that solves the problem.

Teachers' Knowledge

Members of the learning community ask themselves what they should know about knowledge for teaching mathematics to be effective teachers. For that purpose, a member of the community introduced the Knowledge Quartet (KQ) proposed by Rowland et al. (2005) as a theoretical framework for the analysis and reflection processes of their teaching activities. From the perspective of KQ, the knowledge and beliefs that are evidenced in the teaching of mathematics can be typified by means of four dimensions: (1) Foundation, (2) Transformation, (3) Connection, and (4) Contingency.

Analysis of Activities within the Learning Community

The KQ framework is also used as a tool for analyzing the 'Teaching-Reflection Cycle' related to the minimum quadrilateral area problem. For the coding of KQ dimensions, the analysis is based on Rowland, Turner & Thwaites (2014, pp. 319–320) where the contributing codes for each dimension are found. Five videos were analyzed using MAXQDA software to transcribe and for coding.

KQ categories that emerge in an episode of the Teaching-Reflection Cycle

The thoughts that appeared in the reflection process about how Mario taught the HLT were incorporated in Tadeo's teaching. This can be seen as a way to build up practical knowledge for teaching quadratic functions. An example of this statement can be appreciated in the next episode.

In Mario's class, a student passed to the blackboard to solve the problem, assuming that the points P, M, N, S were situated in the midpoint of each corresponding side of the rectangle ABCD. This is considered a situation of deviation from the agenda in the teaching process, an aspect that was not expected, so it is a Contingency (a dimension of the KQ framework). Mario dedicated a long period of time to make the students think about the data of the problem in a different way.

A student in Tadeo´s class, while teaching the HLT makes the same interpretation regarding the midpoints, and drew a rhombus inscribed in the rectangle. Tadeo knew in advance that this could happen and manages to lead students to make the most general interpretation considering the problem's conditions.

Tadeo: Well, but ... is everyone clear about what the problem asks you to calculate?
Student 1: The rhombus area.
Tadeo: Well, here I think, that in the wording there is a part that we are not considering because we put the points P, M, N, and S as he said (referring to student 1), you placed it in the midpoint– [pointing P, M, N, and S for each side of the ABCD rectangle] – (See Figure 1).
Student 1: Ah! M and S are not midpoints.
Student 2: Because it had to be the same distance from A to P and from B to M.
Tadeo: Exactly, you already saw that among the conditions of the problem there is a part that says that the distance from B to M, from C to N, from D to S is the same that from A to P. Did you consider that information?

This is an example of the Foundation Dimension of the KQ Framework. The Contingency situation for Mario became a Foundation situation for Tadeo, due to the previous reflection on the first teaching trial within the community.

Discussion

This is the first 'Teaching-Reflection Cycle' of the community, of several more cycles which are being analyzed as part of a broader research work that aims to observe how these cycles can enrich the practical knowledge of the teacher. This methodology of data analysis supported by the KQ framework can be seen as an appropriate frame for analyzing what happens within the community to show how knowledge is produced.

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