REVIEW OF SLOPE IN CALCULUS TEXTBOOKS

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In this study, we report on how slope is reviewed in a convenience sample of 28 common calculus textbooks published in English. While most calculus textbooks studied included reviews of slope, findings suggest that the reviews are written for students who already have a fairly solid understanding of slope. Slope as a ratio, whether approached visually or nonvisually, serves as a foundational notion for derivative and was the most common conceptualization used in the textbook reviews studied. However, the lack of alternative conceptualizations and connections between various conceptualizations of slope may hinder students deeply understanding other calculus topics. Future study should look at each of these in depth to determine how slope is needed and leveraged when particular calculus concepts are introduced.

Keywords: Calculus, Post-Secondary Education, University Mathematics, Curriculum Analysis

Dietiker (2013) argues that mathematics textbooks can be interpreted as narratives that present mathematical ideas in a purposeful, influential order. Mathematics textbooks create a link between natural language and symbolic mathematical language (Fang & Schleppegrell, 2010), where both languages work using nonvisual elements, such as equations, and visual elements, such as graphs (O'Halloran, 2015). Textbooks play an important role in the way professors shape and sequence their instruction (Davis, 2009) and in how students choose strategies to consider and solve problems (Massey & Riley, 2013). Love and Pimm (1996) have suggested that textbooks are primarily geared toward students. So, textbooks often impact how students learn, aiding students as they form ideas on how to solve problems (Massey & Riley, 2013). Previous calculus textbook research has focused on a) how students consider and solve textbook problems (Lithner, 2003; Lithner, 2004), b) how instructors use textbooks in their teaching (Mesa & Griffiths, 2012), c) how textbooks present and structure examples (Mesa, 2010), d) what is required of students in examples (Özgeldi & Aydın, in press), and e) how representations are coordinated in particular reform textbooks (Chang, Cromley & Tran, 2016).

Slope is a secondary mathematics topic that becomes foundational in post-secondary (i.e., university) mathematics. It plays a key role when contrasting the covariational behavior of linear and nonlinear functions in algebra (Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Teuscher & Reys, 2010) and in the development of derivative in calculus (Zandieh & Knapp, 2006). Research on slope in post-secondary mathematics has increased in recent years even extending into how slope plays a role in multivariable calculus (e.g., McGee & Moore-Russo, 2015). However, examining slope and how it is presented in single variable calculus (henceforth, simply referred to as "calculus") textbooks, has not received attention. Ideally, students should follow and use relations between conceptualizations of slope at will, demonstrating a flexible, integrated understanding of this notion. However, little is known about how, or even if, calculus textbooks review slope. This study considers both calculus for science, technology, engineering, and math (STEM) majors (deemed "STEM textbooks") and applied calculus textbooks often used in classes for business majors as well as life and social science majors (deemed "non-STEM textbooks) to see which conceptualizations of slope are included and if the textbooks are capitalizing on visual approaches in addition to linguistic resources (Moore-Russo & Shanahan, 2014). More specifically, we seek to answer the following three research questions:

1. Is slope reviewed in calculus textbooks? If so, where are slope reviews located?

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2. Which conceptualizations of slope are used in textbook reviews? Are visual or nonvisual approaches to slope taken in textbook reviews?
3. What common links, if any, exist between the conceptualizations of slope reviewed in calculus textbooks?

Literature Review

Importance of Calculus

While a calculus course is required of STEM majors (Bressoud, 2015), applied calculus, without any trigonometry, is often required of business majors as well as social and life sciences majors. Failing, or only marginally passing calculus, is one of the main reasons post-secondary students change their majors (Hensel, Sigler & Lowery, 2008; Kaabouch, Worley, Neubert & Khavanin, 2012; Bressoud, 2015). Many STEM degree programs require a grade of C or higher in calculus to count for credit, with calculus being prerequisite to other courses required in the major, and it is often recommended that students pass calculus at a high level before moving on to further courses (Koch & Herrin, 2006). Engineering students who fail calculus lack the foundation needed for required courses in their majors (Koch & Herrin, 2006; Veenstray, Dey & Herrin, 2008). Student struggles in STEM calculus are not limited to engineering students; studies have shown that calculus attrition rates (i.e., receiving a grade of D or F or withdrawing) for students in physical science or math may be as high as 40% to 50% (Pilgrim, 2010; Fayowski & MacMillan, 2008).

Slope as a Foundational Topic for Calculus

Some mathematics courses follow a vertical path in which certain concepts rely on previous concepts (Treisman, 1992). Many key concepts in calculus build on topics introduced in algebra and precalculus (Habre & Abboud, 2006). In order to develop a robust understanding of foundational ideas in calculus, such as instantaneous rates of change and derivatives, students must first understand average rates of change and the difference between linear and nonlinear functions. Yet, individuals often a) have isolated notions of slope (Dolores Flores, Rivera López & García García, 2019); b) have trouble interpreting different representations of slope (Glen, 2017; Tanışlı & Bike Kalkan, 2018); c) are only able apply slope in particular problem contexts (Byerley & Thompson, 2017); and d) have a limited understanding of linear functions in general, even when able to transition between different representations of linear functions (Adu-Gyamfi & Bossé, 2014).

As students enter post-secondary institutions, the ways they think of slope may be quite limited and different from the ways their professors think of and communicate slope (Nagle, Moore-Russo, Viglietti, & Martin, 2013). Two reasons for this may be related to the limited understanding of slope held by some high school teachers (Coe, 2007; Moore-Russo, Conner & Rugg, 2011; Nagle & Moore-Russo, 2014a; Stump, 1999) and the differences in the way state standards and textbooks address slope (Nagle & Moore-Russo, 2014b; Stanton & Moore-Russo, 2012). No matter why, a lack of prerequisite knowledge often leads to difficulty in understanding later topics, which corresponds with poor performance (Pyzdrowski et al., 2013).

Slope Understanding

Previous studies have considered how slope is characterized in the U.S. and Mexican curriculum (Stanton & Moore-Russo, 2012; Dolores Flores, Rivera López & Moore-Russo, 2020) and conceptualized by a variety of individuals (Moore-Russo, Conner & Rugg, 2011; Nagle, Martínez-Planell & Moore-Russo, 2019; Stump 1999, 2001b). The meaning that an individual makes related to slope, or any other mathematical notion, often depends on what the task at hand evokes (Tall & Vinner, 1981), the representations used to communicate ideas (De Bock, Van Dooren & Verschaffel, 2015) and the individual's prior knowledge or experiences (Vinner, 1992). In short, slope can be conceptualized in many ways, but previous research suggests that both students and teachers often fail to make connections between the various conceptualizations of slope (Coe, 2007; Hattikudur et al., 2011; Hoban, in press; Lobato & Siebert, 2002; Planinic, Milin-Sipus, Kati, Susac & Ivanjek, 2012; Styers, Nagle, & Moore-Russo, in press).

Stump's (1997, 2001a) research findings suggest that teachers rely primarily on ratios as the dominant representations of slope. Even though secondary teachers express concern for students' understanding of slope, they often reduce slope to procedural computations and neglect to regard the importance of developing a conceptual understanding of slope (Stump, 1999). Slope is often reduced to mnemonics that hinder students' understanding of slope as a rate of change (Walter & Gerson, 2007). As a result students enter calculus with isolated notions of slope and are not able to connect slope as a ratio to other ways of conceptualizing slope, such as a measure of steepness (Nagle & Moore-Russo, 2013a; Stump, 2001b). Students are not always able to work with slope in a conceptual way in application tasks (Lingefjärd & Farahani, 2017) nor are they always able to interpret slope in non-standard settings, such as when nonhomogenous coordinate systems are used (Zaslavsky, Sela & Leron, 2002).

Teuscher and Reys (2010) found that while the majority of calculus students could determine over what interval a variable changed by a certain rate, which involved slope, only half of the students were able to determine the interval with the greatest rate of change. They suggested that part of the reason for the difficulty was the vocabulary used in textbooks. Concepts such as steepness, slope, and rate of change are described in different ways among different textbooks, leading to misunderstandings of the questions for some students (Teuscher & Reys, 2010), those who lacked a deep, connected understanding of slope.

Theoretical Framing: Reader-Oriented Theory

Weinberg and Wiesner (2011) wrote that most academic research on textbooks has framed them as static collections of ideas, simply describing students' reading of the texts as extracting information. They sought to characterize the ways in which students interpret textbooks using reader-oriented theory. Reader-oriented theory centers on the idea that meaning of a text is constructed by the reader, not by the text itself. Three ideas about the readers of textbooks emerge from reader-oriented theory: the intended reader, the implied reader, and the empirical reader. The *intended reader* is "the idea of the reader that forms in the author's mind" (Wolff, 1971, p.166, as cited in Weinberg & Wiesner, 2011). The *empirical reader* is the person who actually reads the textbook. The *implied reader* is a concept used to describe the understandings an empirical reader must possess in order to make sense of a mathematics textbook (Weinberg, 2010). Authors should ensure the intended reader and implied reader coincide. This study will use these notions of different readers as a lens to interpret findings.

Methods

The data set consisted of a convenience sample 28 introductory calculus textbooks published in English between 2011 and 2019 that the research team recognized as including the calculus textbooks most commonly used in the United States. These 28 were used since they were available to the lead researcher as sample copies on an electronic book platform through her academic institution. Of the 28, 14 were non-STEM calculus textbooks and 14 were STEM calculus textbooks. In each textbook, the researchers first reviewed the index for any occurrence of the word "slope" to find all instances where slope was reviewed without any calculus content (e.g., limits, derivatives, etc.). All such review instances were included in the study.

Coding

For the current study, Nagle & Moore-Russo's (2013b) slope coding scheme was revised slightly. Five categories were used for the conceptualization of slope. Each of the five categories was divided further in two subcategories to see if the textbook relied on a *visual* or *nonvisual* approach. Table 1 displays descriptions for all of the conceptualization category-approach pairs.

Table 1: Slope Conceptualization and Approach Coding

Conceptualization	Approach	Description
Slope as a *Ratio* (RA)	Visual	rise/run or vertical change divided by the horizontal change
	Nonvisual	$(y_2 - y_1)/(x_2 - x_1)$ or change in y over change in x
Slope as a *Behavior Indicator* of a line (BI)	Visual	line increases, decreases, is horizontal, is vertical (looks like /, \, -, |) for positive, negative, zero, undefined slope respectively
	Nonvisual	line increases, decreases, is constant, or is not a function in other words (i.e., $y_2 > y_1$ for $x_2 > x_1$) for positive slope, (i.e., $y_2 < y_1$ for $x_2 > x_1$) for negative slope, (i.e., $y_2 = y_1$ for $x_2 > x_1$) for zero slope, or (i.e., $x_2 = x_1$ for $y_2 > y_1$) for undefined slope respectively
Slope as denoting *Steepness* of line's angle of inclination with respect to horizontal (ST)	Visual	relates to how inclined, tilted, slanted, or pitched a line is <u>seen</u> as being; greater value of |slope|, line is more steep (i.e., closer to vertical); closer to zero value of |slope|, line is less steep (closer to horizontal); since horizontal lines have no tilt, they have zero slope
	Nonvisual	relates to how extreme a line is <u>calculated</u> as being; the greater the value of |slope|, the more steep the line over an interval (e.g., the closer to infinity the value of $\lvert y_2 - y_1\rvert$ is); the closer to zero the value of |slope|, the less steep the line over an interval (e.g., the closer to zero the value of $\lvert y_2 - y_1\rvert$ is); horizontal lines have $\lvert y_2 - y_1\rvert = 0$ for all y values; so, slope is zero
Slope as a *Constant Parameter* (CP)	Visual	emphasis on the uniform "straightness" of the line's entire graph; no matter which segment of the line is considered, the straight slope remains the same between any two points due to similar triangles
	Nonvisual	emphasis that a single constant holds a property for the line's equation/table (not dependent on input); for any interval of a line, slope calculations remain the same between any two points
Slope as *Determining Relationships* between lines (DR)	Visual	two unique lines have the same slope if and only if they never intersect in two-dimensions (i.e., are parallel); two unique lines have different slopes if and only if they intersect at a common point; two unique, nonvertical lines have negative reciprocal slopes if and only if their intersection is at a right angle
	Nonvisual	two unique lines have the same slope if and only if a system of these two lines has no solution; two unique lines have different slopes if and only if the system of these two lines has one solution; two unique, nonvertical lines have negative reciprocal slopes if and only the product of their slopes is -1

A textbook was used as a unit of analysis and coded as an entity, meaning that if a textbook had more than one instance of one of the 10 categories (i.e., *visual* and *nonvisual* approaches to a *Ratio*, *Behavior Indicator*, *Steepness*, *Constant Parameter*, and *Determining Relationships* conceptualization), it was only marked once. To explain coding, consider this example involving two different approaches to the same conceptualization category. For example, consider the numerical computation of slope between two points (coded RA_n, for *Ratio-nonvisual*) being accompanied by a graph with labeling of Δy for the vertical displacement and Δx for the horizontal displacement (coded RA_v, for *Ratio-visual*). This graph of a line with Δy and Δx labeled and accompanied by $m = \frac{\Delta y}{\Delta x}$ would be coded as the link RA_v-RA_n. The implied reader, in this case, is meant to have an understanding of the slope computation and how it correlates with the visual markings indicated on the graph. Similar coding was used if two different conceptualization approaches were linked.

Findings and Discussion

Research Question 1: Slope Reviews in Calculus Textbooks

Since the concept of slope is an important building block for students taking calculus (Noble, Nemirovsky, Wright & Tierney, 2001), it was not surprising that all but one of the STEM textbooks in this study contained at least some review of slope. Most (22 of the 28) textbooks had the slope review at the beginning of the textbook only, while 3 had some slope review at the beginning and at the end of the textbook. This suggests that most calculus textbook authors feel that a review of slope should be available to students (or covered through instruction) prior to the introduction of derivatives and other calculus concepts. Calculus textbook authors appear to envision intended readers as students who need to have a solid base of prerequisite knowledge that includes an understanding of slope.

Research Question 2: Slope Conceptualizations in Calculus Textbooks

We now consider which conceptualizations of and approaches to slope were used in the sample of textbooks used in this study focusing on the implied reader to consider how concepts emphasized in different textbooks' slope reviews involve different understandings readers must possess in order to make sense of the calculus concepts presented in the textbooks. Table 2 displays the findings from the textbooks. All five of the conceptualizations of slope were used in at least one of the textbooks.

Almost all (25 of 28) textbooks used the *Ratio* conceptualization of slope. Postsecondary instructors and calculus students have been found to respond to open-ended questions about slope with responses that included a *visual* or *nonvisual* approach to *Ratio* (Nagle et al., 2013), as have high school teachers (Stump, 1999). So, this finding suggests that the slope reviews in the textbooks might be trying to connect with how students often think of slope. Understanding slope as a *Ratio* is important in calculus, especially in the understanding of derivative, when the limit of a difference quotient ties together the concepts of slope, limit, and derivative. So, it is not surprising that *Ratio* is the most prevalent conceptualization in calculus textbooks.

For calculus, students need a solid understanding of the idea of ratio (not just in the sense of slope, but in general) that goes beyond chanting "rise over run" or plugging and chugging into a formula. This is important since research (e.g., Carlson, Madison, & West, 2015) has shown that students often do not consider slope as representing the ratio of two covarying quantities in complex problems, such as those found in calculus. Textbook authors who do not consider this may lead to a disconnection between the intended and implied readers. In calculus, readers need to understand that the visually-oriented, rise-to-run graphical comparisons of a linear segment and the corresponding algebraic formulas that represent the slopes of secant lines connecting two points on a curve approach the value of the slope of a tangent line to the point on a curve in order to understand how a derivative is defined. However, in their reviews of slope, most textbook authors did not mention that slope is a foundational topic for understanding calculus concepts, and focused solely on finding slope as a numerical value (*nonvisual*) or as a property associated with the image of a line (*visual*).

Table 2: Slope Conceptualizations and Approaches in Textbooks

Conceptualization	Approach	Textbook Type		Total
		non-STEM	STEM	
Ratio (RA)	Visual Only	0	0	0
	Nonvisual Only	0	0	0
	Both	14	11	25
Behavior Indicator (BI)	Visual Only	3	7	10
	Nonvisual Only	1	0	1

	Both	10	4	14
Determining Property (DP)	Visual Only	1	0	1
	Nonvisual Only	1	2	3
	Both	9	7	16
Constant Parameter (CP)	Visual Only	2	2	4
	Nonvisual Only	1	1	2
	Both	6	7	13
Steepness (ST)	Visual Only	2	3	5
	Nonvisual Only	0	0	0
	Both	6	2	8

Behavior Indicator tied as the most used conceptualization of slope in the reviews, present in 25 textbooks (as was the *Ratio* conceptualization). However, all *Ratio* coded textbooks used both *visual* and *nonvisual* approaches, while *Behavior Indicator* textbooks did not. STEM textbooks more often used *visual* approaches only. Slope is often associated by students visually as the way a line is displayed on a graph (Moore et al., 2013). In Nagle and colleagues' 2013 study, this was a conceptualization commonly reported by post-secondary calculus students but not by their instructors. In the cases of textbooks emphasizing *Behavior Indicator*, the implied reader should be able to build an understanding the ideas of "increasing" and "decreasing" as well as related conventions of graphing to slope. In order to understand the idea of derivative and related topics (e.g., role that a slope of zero has in identifying potential relative extrema in the first derivative test), students will need to know more than just how the slope of a tangent line behaves visually; they will need to be able to work with intervals of functions using formulas.

Determining Property was the conceptualization used third most by textbook authors. In the textbooks where this conceptualization was noted, implied readers should be able to build on an understanding of concepts such as parallel, perpendicular, reciprocal, and so on. In the case of textbooks using this conceptualization, readers are typically asked to interpret two or more lines that are being compared (either graphically or using formulas). This should help prepare readers for tasks involving identification of the equation of a normal line, which is perpendicular to the line tangent to a curve at a point. Students also need to know that parallel lines have the same slope in order to understand the Mean Value Theorem and Rolle's Theorem.

The second least used slope conceptualization was *Constant Parameter*. Textbooks which include this conceptualization require their students to understand what the word "constant" means in a mathematics context for linear functions where slope acts as a parameter that results in a constant numerical change seen in tables or in the graphical straightness noted in a *visual* display of a line. Implied readers need to leverage an understanding of the straightness of a line for approximations over sufficiently small intervals when using linearization.

Steepness was the least used conceptualization of slope used in the calculus textbooks. Students should be able to construct ideas of "steepness" in a physical sense that relates the higher the absolute value of the slope of a line is, the steeper that line is. This understanding is often needed for related rates problems involving angles of inclination and right triangles, such as tasks involving where the vertical rate of ascension for a rising balloon is given.

Research Question 3: Links between Slope Conceptualizations

To answer the third research question, we now consider common links between the slope conceptualizations in the textbooks. In order to be coded as a link, the textbook had to indicate that two conceptualization-approach pairs related to the same idea. Table 3 displays the type of links present in the textbooks, and the total number of textbooks per link type. The RA_n-RA_v link was most frequent (22), which supports that the implied reader must to be able to relate the *visual* and *nonvisual* approaches of the *Ratio* conceptualization in order to understand slope. This was frequently shown as the graph of a line with the rise and run indicated, accompanied by the corresponding numerical calculation. Table 3 displays most frequent links, those that occurred in at least 10 textbooks.

Table 3: Links between Conceptualization-Approach Pairs in Textbooks

Link	Textbook Type		Total
	non-STEM	STEM	
RA_n-RA_v	12	10	22
CP_n-RA_n	9	6	15
DP_n-DP_v	7	7	14
CP_v-RA_n	7	6	13
BI_n-BI_v	8	4	12
CP_n-CP_v	6	6	12
CP_n-RA_v	6	6	12
CP_n-RA_v	5	6	11

Links containing either the *Ratio* or *Constant Parameter* conceptualization (with either a *visual* and *nonvisual* approach) were the most frequent. Given the prevalence of the *Ratio* conceptualization in textbooks, it is not surprising that its *nonvisual* and *visual* approaches would be linked most often.

Research (e.g., Nagle & Moore-Russo, 2013b) has referred to slope as a constant ratio, whereas *Constant Parameter* and *Ratio* were defined separately in this study. As such, the links containing either the *Ratio* or *Constant Parameter* being the most common is not surprising. In other words, understanding that slope is a constant rate of change between two covarying quantities, an equivalence class of ratios appears to be considered by authors as pivotal when first learning the concept of derivative. This suggests that the implied readers typically will need to make the link that slope can be considered as both a *Ratio* and a *Constant Parameter*.

The *Behavior Indicator* conceptualization occurred just as often as the *Ratio* conceptualization in 25 of the 28 textbooks. However it was not linked to other conceptualizations of slope very frequently. It does not seem that textbook authors deemed this way of thinking about slope to need to be connected to other ways of thinking of slope. This lack of connection could lead readers to concentrate on procedures without a connected, conceptual understanding of why first derivative tests are used to determine functional behavior in calculus.

Conclusions, Instructional Implications, Further Study

Slope is not heavily reviewed in calculus textbooks. Given the importance of understanding slope for calculus and how textbooks review slope, textbook writers seem to be assuming that the intended readers of these texts have a healthy understanding of slope upon entering calculus. Some conceptualizations of slope are sparsely represented in textbooks, and many textbooks do not provide a well-rounded, connected review of slope. Instructors must be aware that they may need to provide additional review over what is offered in textbooks to ensure that students are making connections between different conceptualizations of slope so that students have the robust understanding of slope that is needed as a foundation for the topics they encounter throughout calculus. Instructors should read the slope reviews, encourage students to do the same, and then be aware of other conceptualizations of and approaches to slope that they may need to provide to their students that are not present in textbooks.

The emphasis on slope as a *Ratio* and linking this idea to slope as a *Constant Parameter* should prepare students for the limit definition of a derivative; however, the lack of connections between the various conceptualizations of slope may result in students' failure to deeply understand other calculus topics that require alternative notions of slope. The role of slope in introducing derivatives is documented, but it is important that instructors also consider how textbooks leverage slope to introduce other calculus topics, such as how students come to think about average rates of change, why a derivative of zero may yield relative extrema, how parallel lines play a role in the Mean Value Theorem, how the "straightness" of a line is leveraged in linearization, etc. Future study should look at each of these in depth to determine how slope is needed and leveraged when particular calculus concepts are introduced.

One limitation of this study is that it only considered the stand-alone reviews of slope. It is possible that textbook authors are using a just-in-time review approach and connecting different conceptualizations of slope while introducing the calculus concepts themselves. Future research should consider this possibility.

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