PROSPECTIVE ELEMENTARY TEACHERS' CONTENT KNOWLEDGE OF DECIMAL MAGNITUDE AND PLACE VALUE

Rachel Starks
Boston University
RNStarks@bu.edu

Ziv Feldman
Boston University
ZFeld@bu.edu

Research suggests that robust mathematical knowledge for teaching is essential for high quality instruction and learning, and yet studies also reveal that prospective teachers (PTs) may not have had sufficient opportunities to develop this knowledge. Decimal understanding is one particular area of difficulty for elementary students and PTs alike, but few studies have focused on characterizing PTs' decimal understanding. In this study we examine 28 PTs' ability to create and explain models for comparing decimals, following instruction on this topic. We find that participants are able to effectively use models to identify and reason about the larger of two decimals, but that they struggle to articulate underlying mathematical ideas such as the role of place value in decimal magnitude or connections among decimal models.

Keywords: Teacher Education – Preservice, Mathematical Knowledge for Teaching, Rational Numbers

Research suggests that teachers of mathematics must have substantial knowledge of the content that they will teach and of appropriate ways to do so; these together are known as mathematical knowledge for teaching (MKT; Ball, Thames, & Phelps, 2008). Unfortunately, some studies also suggest that prospective elementary teachers' (PTs') MKT is still developing; areas for growth span topics such as fractions (e.g., Van Steenbrugge et al., 2014), geometry (e.g., Aslan-Tutak & Adams, 2015), decimals (e.g., Stacey et al., 2001; Widjaja et al., 2008) and more (Hill, 2010). Although decimal concepts have been shown to be difficult for learners, fewer studies focus on PTs' knowledge of decimals (Kastberg & Morton, 2014) than fractions (Olanoff et al., 2014) or whole numbers and operations (Thanheiser et al., 2014). This study attempts to contribute to and update the small body of literature on PTs' knowledge of decimal magnitude and place value. Based on PTs' responses to two open-ended tasks, we describe the models and strategies they use to make sense of and compare two decimal quantities. Further, we analyze PTs' written explanations of their models and strategies, and the underlying mathematical ideas that they identify as important.

Background & Theory

Here, we introduce some recommendations for supporting PTs' MKT and studies which give images of it, then highlight a gap in this literature. We describe how the MKT framework bounds our study by defining what is visible in the data, and discuss *conceptual understanding* and how this strand of mathematical proficiency figures in our analysis.

Decimals in Elementary Mathematics and Teacher Preparation

Elementary mathematics standards span many topics, including *number concepts*. Number concepts pertain to the structure of the base ten system and its extension to decimal quantities. Number concepts are important because of the ways in which they undergird foundational elementary mathematics such as counting and operations (Association of Mathematics Teacher Educators, 2017; Conference Board of the Mathematical Sciences (CBMS), 2012). Knowledge of decimals, specifically, is also expected of students. In grades 4 and 5 alone, the Common Core State Standards call for students to order, compare, and model decimals (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

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Because of their presence in elementary school mathematics, policy documents call for coursework for PTs to also focus on number concepts, including decimals (CBMS, 2012). This call is amplified by empirical research suggesting that children and PTs alike struggle with decimal understanding (e.g., Muir & Livy, 2012, Steinle & Stacey, 1998). Stacey and colleagues (2001) found that some overgeneralizations common among children are much rarer among PTs, but that other misconceptions persist from childhood through to adult populations. Despite knowledge that sense-making around decimals is challenging for PTs, few studies have characterized their understanding. In Kastberg and Morton's (2014) literature review, they identified just three studies since 1998 which attended specifically to PTs' decimal understanding. Broader inclusion criteria uncover a few additional studies with relevant findings, but PTs' decimal understanding and learning processes remain under-researched. The literature does not effectively characterize PTs' understanding of the magnitudes of decimal quantities or explore their reasoning related to comparing decimals.

It is concerning that research on PTs' decimal MKT is so scarce. Without robust images of PT knowledge, mathematics teacher educators may be without the information needed to support PTs' growth. PTs must have deep understanding of decimal concepts and procedures, since this topic is prevalent in upper elementary curriculum, and since their MKT is known to contribute significantly to quality teaching and learning (Hill et al., 2005, 2008). We turn our focus to a brief elaboration on the construct of MKT generally, and a look at how it informs this study.

Mathematical Knowledge for Teaching

MKT includes *subject matter knowledge* and *pedagogical content knowledge* (Ball, Thames, & Phelps, 2008). The former is knowledge which a teacher must have about the mathematics itself, including knowledge which is common among adults, as well as specialized knowledge, needed primarily or exclusively by teachers. Pedagogical content knowledge focuses on the teaching of mathematics. Our participants had the opportunity to demonstrate understanding of the relative sizes of two decimals, and knowledge of how tools for modeling decimals relate to concepts about decimal place value. These are examples of subject matter knowledge; PTs in this study did not have the opportunity to display pedagogical content knowledge.

Conceptual Understanding as Part of Mathematical Proficiency

Given the importance of teachers having robust MKT, policy documents recommend that prospective elementary teachers engage in substantial coursework focusing specifically on elementary mathematics (CBMS, 2012). Studying these concepts at an appropriately deep level for adults who are future educators involves a higher level of connection-making between mathematical ideas than would be expected of elementary students, in part because robust MKT includes knowledge of the *connectedness* of mathematical ideas within and across grade levels. Furthermore, this connection-making is important because it is characteristic of *conceptual understanding*, one of the five "Strands of Mathematical Proficiency" outlined by the National Research Council (2001). Mathematical proficiency requires learners to have *well-connected* knowledge of concepts within a larger body of mathematical knowledge. This conceptual knowledge is, by nature, "rich in relationships" (Hiebert & Lefevre, 1986, p. 3). In our study, we attend to the mathematical connections that participants in our data set do or do not make.

Research Questions

In this study, we pose two main research questions. Following instruction on elementary decimal concepts, (1) what models and other strategies do PTs use to compare two decimal quantities, and to what extent do they use these models appropriately and successfully? (2) What is the nature and quality of PTs' understanding of decimal place value and magnitude, as evidenced by their writing about comparing two decimal quantities?

Methods

Data for this study was collected in the context of the Elementary Mathematics Project (EMP), an NSF-funded project which designs and conducts research surrounding curriculum for use in content courses for prospective elementary teachers. One of EMP's seven instructional units is *Number Concepts*, which focuses on the consistencies of place value structure from large to small (decimal) numbers, as well as on modeling numbers. Regarding decimals, PTs using this curriculum have the opportunity to learn about decimal place value; area, linear, and other models; comparing decimals by place value or by looking at same-sized pieces; and more.

Participants in this pilot study are 28 PTs from two different institutions. Site A was a public community college in the Northwest of the United States. The course instructor at Site A has a doctorate in math education, but her appointment is in the mathematics department; she taught 17 of the 28 participants. Site B was a private four-year college in the Midwest. The instructor has a master's degree in math education but is also housed in a mathematics department. Socio-demographic data was not collected from participants, however, the student body of undergraduate teacher education programs tends to be primarily female, and roughly 19-22 years of age. All participants used the EMP Number Concepts unit, then completed an eight-item, open-ended post-test, designed by the EMP team. We analyzed this item:

> As a future teacher, you may encounter a student who is having difficulty determining which of two decimal values is greater. For example, 0.4 and 0.32.
>
> a) Provide a model that would help a student to think about the sizes of 0.4 and 0.32.
>
> b) Explain how your model would help a student compare these two quantities and which important mathematical ideas it addresses.

We analyzed responses by first open coding all elements of PTs' drawings, writing, and symbols. We did this by examining whether the participant explicitly and correctly identified the larger value, what model they provided and how it was labeled, and what they wrote about. This resulted in 32 codes which together captured PTs' choices of model and the content of their explanations. 100% consensus was achieved between two coders, after discussion.

Preliminary Results

Promising findings from our preliminary analysis include the fact that the majority of PTs explicitly identified the correct quantity as larger (n=20, 71%) and were able to provide one or more models that was accurate and useful for comparing (n=25, 89%). Seven responses were unclear as to whether 0.4 or 0.32 was larger, but only one was explicitly incorrect. Ten PTs provided decimal squares only as a model for comparison, nine provided number lines only, and six gave both. (It is unsurprising that these models were most common, in that these were two of the most prominent models in the EMP curriculum.) Two of the three PTs remaining used place value charts. This reveals that, generally, PTs are able to compare decimals, and to create and interpret models to aid in comparison, following instruction.

Data from this study also uncovered three primary challenges and areas for growth for PTs. First, we found that it seems to be more difficult to use a number line than decimal squares for the purposes of understanding decimal magnitude and relative magnitude. Most decimal squares were proportional and well-labelled, showing the size of each of the two decimal values, relative to a whole, and to each other. PTs' number lines were also generally proportional (80% of 15 number lines) and showed how hundredths could be created by partitioning tenths into tenths (67%). However, many number lines were truncated (67%), often beginning at 0.3, which limited their ability to communicate the magnitude of each of the individual decimal quantities. Furthermore, two of the 15 number lines were partitioned into elevenths instead of tenths, and two were drawn or interpreted "backwards" (smaller numbers to the right); comparable challenges did not emerge for decimal squares. Finally, explicit interpretation of the number lines was rare, just 27% described how their number line should be read and understood, while 63% of PTs who drew decimal squares explained how to interpret their model.

Second, we found that PTs' explanations did not always attend to relevant connections or reasoning, specifically surrounding the representation of 0.4 as 0.40. Of the 28 responses, 11 PTs stated or showed that 0.4 is equivalent to 0.40, or that it is appropriate to "add a zero" to the end of a decimal number. However, only five of these 11 PTs explained why this is true or useful. Though several PTs highlighted this equivalence or stated this "trick" for re-representing the quantity, less than half of those who did so attempted to justify the equivalence, or explain why a learner might find it easier to think of four tenths as forty hundredths. This leaves us uncertain as to the depth of understanding achieved by some of these PTs.

Finally, we found that references to the importance of place value were conspicuously rare. Although PTs had been asked to "Explain how your model would help a student compare these two quantities and which important mathematical ideas it addresses," less than half of the participants mentioned place value as one of these important mathematical ideas. This was highly surprising to us, as we conceive of place value as *the most* important mathematical idea undergirding these models and comparisons.

Discussion

Above, we highlighted the importance of well-connected *conceptual understanding* for both students and teachers. Our findings suggest that, while PTs have notable strengths for completing decimal tasks and using relevant tools to do so, they are less likely to articulate underlying mathematical connections. For example, few PTs in our study connected their models to decimal place value concepts, or their strategies for comparison to reasoning and justification for those strategies. This calls into question whether they have sufficient conceptual understanding to contribute to robust MKT.

A clear vision of PTs' skills and knowledge related to decimal concepts and procedures is useful and necessary for mathematics teacher educators, as they are charged with developing curriculum for use in teacher preparation coursework. We suggest that characterizations such as we have provided here may support these teacher educators in understanding PTs' strengths and needs, a first step in making changes to improve teacher preparation courses.

Next steps for this study include continuing analysis of a larger set of tests from the corpus of EMP data. The 28 tests in this study were selected as a pilot sample, but represent only about 10% of the PT participants who took the EMP unit test during this phase of data collection. We also hope to analyze corresponding pre-tests, to better understand the growth which may have happened as a result of engagement with instruction around decimal concepts. In addition to going broader, we also hope to go deeper by re-examining the types of claims that PTs made about place value in particular and exploring possible connections between these claims and PTs' chosen models. This will empower us to create more robust characterizations of PTs' knowledge of decimal place value concepts and examine the ways in which models may facilitate or demonstrate knowledge development.

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