AN ANALYSIS OF COORDINATE SYSTEMS PRESENTED IN GRADE 6-8 TEXTBOOKS

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We analyzed a total of 1129 tasks in grade 6–8 textbooks to examine the type of coordinate system presented and the associated graphing activity required in each task. We share our findings and discuss educational implications of such findings.

Keywords: Representations and Visualization, Coordinate System, Graph, Textbook Analysis

Middle school is a critical time for students to develop robust understandings of coordinate systems and graphs. In the Common Core State Standards for Mathematics (CCSSM; NGA Center & CCSSO, 2010), for example, students are first introduced to the Cartesian plane and learn to plot or interpret points in the first quadrant in 5th grade; thereafter, students are expected to use the Cartesian plane for exploring and representing other mathematical ideas including geometrical shapes, proportional relationships, number systems (6-7th grade), and graphs of linear relationships (8th grade). In this report, we share results from an analysis on the types of coordinate systems presented and associated graphing activity required in grade 6–8 textbooks and discuss educational implications of such findings.

Conceptual Framework

By coordinate system, we mean a representational space in which an individual systematically coordinates quantities (Thompson, 2011) to organize some phenomenon. We have previously distinguished between two types of coordinate systems depending on the goal they serve: spatial and quantitative coordinate systems (Lee, Hardison, & Paoletti, 2018; 2020). A spatial coordinate system is used to quantitatively organize a space in which a phenomenon is situated (Figure 1). Constructing a spatial coordinate system involves an individual organizing a space by (mentally) overlaying a coordinate system onto some physical or imagined space being represented where objects within that space are tagged with coordinates. For example, in Figure 1b a coordinate system is overlaid onto a region of a city from a bird's eye view where the *x*- and *y*-axes coincide with roads in the city.

On the other hand, a quantitative coordinate system is used to coordinate sets of quantities by constructing a geometrical representation of the product of measure spaces (Figure 2). Constructing a quantitative coordinate system involves an individual extracting quantities from the space in which a phenomenon occurs and projecting them onto a new space, different from the space in which the quantities were originally conceived. For example, in Figure 2b the coordinate system is showing the relationship between time (in minutes) and the number of boxes a machine packages over time where both quantities were taken from a space separate from the presented coordinate system.

Relatedly, graphs represented in each of these coordinate systems are fundamentally different (Lee, Hardison, & Paoletti, 2018; 2020). Graphs created on spatial coordinate systems can be viewed as projections or traces of physical objects or phenomena onto an analogous space containing the original objects or phenomena. Whereas, in a quantitative coordinate system, graphs are not projections of physical objects or phenomena from the same space containing the original objects or phenomena. Due to this distinction, different ways of reasoning could be productive when creating and interpreting spatial and quantitative coordinate systems and their associated graphs (c.f., Lee, Hardison, Paoletti, 2020).

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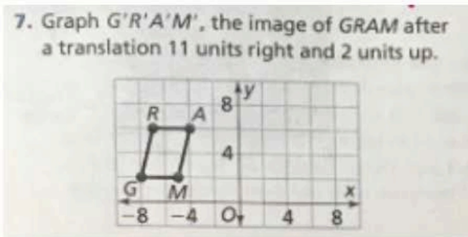
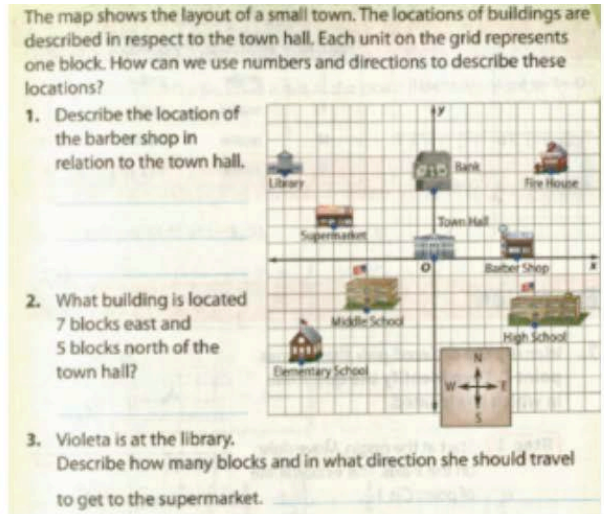
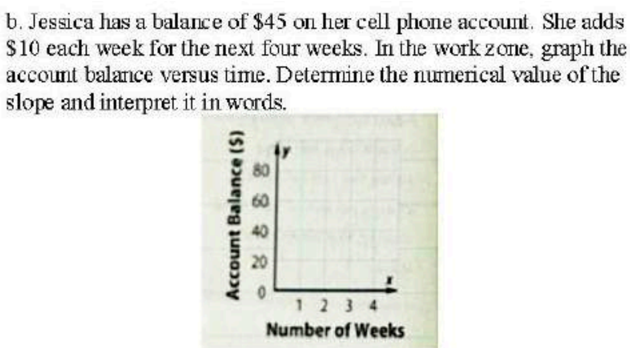


(a) Task from Berry III et al. (2017f), p. 301.

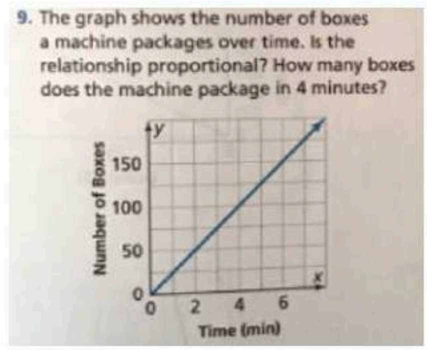


(b) Part of a task from McGraw-Hill Education. (2015a), p. 85.

Figure 1. Examples of tasks using a spatial coordinate system.



(a) Task from McGraw-Hill Education. (2015d), p. 464.



(b) Task from Berry III et al. (2017c), p. 119.

Figure 2. Examples of tasks using a quantitative coordinate system.

Using this framework, our goal in this study was to investigate the different types of coordinate systems presented in grade 6–8 textbooks. Specifically, we examined (a) what type of coordinate systems textbooks present, (b) what type of graphing activities problem solvers are prompted to engage in, and (c) how frequently these coordinate systems and graphing activities appear in textbooks. We emphasize that the textbook analysis involved *our* interpretations of the textbook author's intended use of coordinate systems, which do not necessarily coincide with how students might perceive of the coordinate system.

Methods

We extracted and coded a total of 1129 tasks from three major textbook series (*Mathematics in Context*, *enVision Math 2.0 Common Core,* and *Texas Math TEKS*; see references) for grades 6–8 to date. The three series were selected to represent a variety of curricula. The first step in our analysis involved selecting and extracting tasks. The criterion for inclusion was that the task presented a pre-constructed two-dimensional coordinate system, either left blank or containing a graph. Relatedly, we also included tasks referring to previous tasks containing a pre-constructed coordinate system. We excluded tasks that had coordinate system-like grids but did not explicitly involve problem solvers to attend to the coordination of quantities (e.g., a grid superimposed onto a shape to find its area, box-and-whisker plots, and bar graphs with categorical data). Our unit of analysis, what we call a task, is a sequence of explanations or questions surrounding a single context or a single coordinate system. This means that explanations/questions about a single context with several coordinate systems (e.g., comparing several graphs) and explanations/questions with multiple contexts around a single coordinate system (e.g., graph several things on the same coordinate system) all counted as a single task.

After extracting tasks, we coded each task along two dimensions. The first dimension is the type of coordinate system. A task received the code *spatial* if the coordinate system is spatial; *quantitative* if the coordinate system is quantitative; *both* if the task involves both types of coordinate systems in a single task (e.g., comparison tasks with both types); and *neither* if it was difficult to discern as spatial or quantitative due to lack of context (e.g., *x-y* graph without specification of what *x* and *y* represent). When necessary, we referred back to previous or subsequent tasks in the textbook to determine the context of the task.

The second dimension was the type of graphing activity required in the task. A task received a *create* code if it requires problem solvers to create a graph by plotting a point or collection of points (e.g., tasks in Figures 1a and 2a). A task received an *interpret* code if it requires problem solvers to make sense of a pre-constructed graph, such as describing the relationship between two variables or constructing an algebraic equation that describes the graph (e.g., tasks in Figures 1b and 2b). Our distinction between create and interpret is similar to Leinhardt et al.'s (1991) distinction between construction and interpretation; however, different from Leinhardt et al., we consider building algebraic functions for a graph as interpretation and not construction. A task received a *both* code if it required the problem solver to both create and interpret a graph; a *neither* code if there were no requirement for the problem solver to create or interpret a graph.

We reiterate that codes were attributed to each task based on our interpretations of the authors' intention of the task. Once all tasks were coded, we compared our codes and when there was a disagreement, we discussed them to come to a consensus. Finally, we recorded the mathematical topic covered in each task using the topic names used in the textbook.

Findings and Discussion

The results are summarized in Table 1. Because our purpose was not to compare textbooks, we report only on the total frequencies for each code across all textbooks within each grade. As shown in Table 1, the coordinate systems were predominantly *quantitative* and most tasks required problems solvers to *interpret* a graph. There were consistently and predominantly more tasks that required students to interpret a graph rather than create a graph across all three grades. However, the trend in coordinate system type changed across grade levels. In grade 6, both types appeared close in frequency. Many grade 6 textbooks introduced the Cartesian plane, asked students to plot points or enact operations on coordinates (i.e., coordinate geometry) and then used the Cartesian plane to represent graphs of functions. As such, coordinate systems were often first introduced spatially and students were expected to transition from a spatial to quantitative coordinate system unproblematically. In grade 7, there was a stark difference in the number of quantitative coordinate systems (n=128) in comparison to those that were spatial (n=8). Relatedly, in grade 7 textbooks we coded, statistical graphs (e.g., bar graphs, histograms) and linear relationships were the main focus of content. Finally, in grade 8, there was a more balanced use of quantitative and spatial coordinate systems, with more quantitative (58%) than spatial (42%). In grade 8, textbooks covered linear relationships (functions, systems of equations) and statistical graphs in which quantitative coordinate systems prevailed; however, they also covered topics such as transformations of shapes and the distance between points, in which spatial coordinate systems were used.

Table 1. Number of Tasks by Problem Type, Coordinate System Type, and grade level.

Grade	Problem Type	Coordinate System Type				Total
		Quantitative	Spatial	Both	Neither	
6	Create	10	0	0	7	17
	Interpret	42	51	0	15	108
	Both	18	12	0	3	33
	Neither	0	0	0	1	1
	Grade Total	70	63	0	26	159
7	Create	9	0	0	15	24
	Interpret	83	3	0	19	105
	Both	35	5	0	4	44
	Neither	1	0	0	5	6
	Grade Total	128	8	0	43	179
8	Create	27	22	0	34	83
	Interpret	225	151	0	124	501
	Both	84	71	0	44	199
	Neither	0	0	0	8	8
	Grade Total	336	244	0	210	791
Grand Total		534	315	0	279	1129

Note there were no tasks that involved both types of coordinate systems but a total of 279 tasks coded neither for coordinate system type. Previously, Paoletti et al. (2016) analyzed graphs in STEM textbooks and practitioner journals at the undergraduate level and found that the majority of graphs either mathematized a spatial situation or represented two (contextual) quantities. On the other hand, they found that most graphs in commonly used precalculus and calculus mathematics textbooks represented two decontextualized quantities, finding a discrepancy between graphs students experience in their math classes and those used in other STEM fields. Looking across our *interpret* graph tasks, 22.13% of those tasks used decontextualized coordinate systems, thus received a *neither* coordinate system type code. Although not as dramatic as Paoletti et al.'s findings, the mathematics textbooks we analyzed also presented decontextualized graphs for students to interpret, which increased over time (15, 19, 124 tasks in grades 6, 7, 8, respectively).

Based on our findings, we propose the following changes in curricula (and relatedly, in teaching) on coordinate systems and associated graphs to support students' mathematical development as well as potential future STEM courses and careers: (a) tasks drawing from contexts that afford opportunities to develop a balanced understanding of both coordinate system types; (b) a more balanced graphing activity associated with coordinate systems, and hence more opportunities for students to create graphs; and (c) better support in curricula materials assisting students' transitions from spatial to quantitative coordinate systems.

In this study, we specifically focused on tasks that presented pre-constructed coordinate systems. Future research directions include identifying the different types of activities required for coordinate systems (e.g., create or interpret a coordinate system) in conjunction with the extant three dimensions (coordinate system type, graphing activity type, and mathematical topic).

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