LEARNING TO POSE PROBLEMS WITHIN DYNAMIC GEOMETRY ENVIRONMENTS: A SELF STUDY INVOLVING VARIGNON'S PROBLEM

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This paper reports my second experience on my trajectory to learn how to pose mathematical problems within Dynamic Geometry Environments. I used The Geometer's Sketchpad and mathematical reasoning as tools to verify the plausibility and reasonability of each new problem situation. Using a problem-posing framework that I had developed during my first problem-posing experience within dynamic geometry environments, and subsequently refined and enriched with subsequent tasks, I was able to generate a diversity of problems by modifying the attributes of Varignon's problem. Among the problems generated were special problems, general problems, extended problems, further extended problems, converse problems, and proof problems. Examples of each of these types of problems are provided.

Keywords: Problem posing, problem solving, teacher educators, technology

Engaging in problem-posing tasks is recognized by mathematicians (e.g., Halmos, 1980; Polya, 1945/1973), mathematics educators (Brown & Walter, 1983, 1993; Kilpatrick, 1987; Silver, 1994, 2013), and professional organizations (Australian Education Council, 1991; National Council of Teacher of Mathematics [NCTM], 1989, 1991, 2000) as a worthwhile mathematical activity. According to Halmos (1980), the heart, the essence, of mathematics consists of problems. NCTM (1991), on the other hand, calls for all students to "be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95).

Purpose of the Study

Problem posing continues to receive increased attention from curricular, pedagogical, and research perspectives as attested by the recent publications of two books: *Mathematical problem posing: From research to effective practice* (Singer, Ellerton, & Cai, 2015) and *Posing and solving mathematical problems: Advances and new perspectives* (Felmer, Pehkonen, & Kilpatrick (2016). Initially, most research focused on understanding and documenting students' abilities to pose mathematical problems (Ellerton, 1986a, 1986b, 1988; English, 1996, 1997, 1998, 2003; Silver & Cai, 1996). If teachers and prospective teachers are to engage their students in problem-posing activities, it is important that they have experiences in problem generation. To help students to enhance their problem-posing abilities, research also examined teachers' approaches to pose mathematical problems (Author, 1998; Crespo, 2003; Ellerton, 2013; Engström & Lingefjärd, 2007; Lavy & Shriki, 2010; Silver et al. 1996). However, as noticed by Beswick and Goos (2018) and Castro Superfine and Li (2014), mathematics teacher educator knowledge has received limited attention.

While numerous studies on problem posing have investigated both students and teachers' abilities to pose problems, little research has been done on mathematics teachers educators' abilities to pose mathematical problems. I extend this research on problem posing by focusing on myself as teacher educator, a teacher of teachers. As noted by Suazo-Flores et al. (2019), qualitative methodologies such as narrative inquiry, self-study, and autoethnography have increasingly becoming modes of inquiry in mathematics teacher education research.

The purpose of this paper is to describe the types of problem that I have generated by modifying the conditions of Varignon's problem. To understand how I came to pose the problems, I present a brief

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story of my experiences with a problem-posing framework and how it enhanced my abilities to pose mathematics problems with the support of The Geometer's Sketchpad (GSP).

Perspectives on Mathematical Problem Posing

Problem posing tasks involve both the generation of new problems aimed at exploring and examining a given situation, as well as the reformulation of given problems (Silver, 1994). As noted by Silver (1994), problem posing can occur before, during, and after solving a given problem.

When we are trying to solve a challenging problem, a strategy is to reformulate the problem into an equivalent problem to make it more accessible. For example, we could reformulate a geometric problem in terms of algebra. A second way to reformulate a problem is to "think of a related, more accessible problem" (Polya, 1945/1973).

Problem posing can also occur before and after problem solving. It can occur before problem solving when the goal of the task is not to solve a mathematical problem, but to simply create new mathematical problems. It can occur after solving a problem as we examine the problem and pose follow-up questions or problems, a stage in the problem-solving process coined "looking back" by Polya. Brown and Walter (1983, 1993, 2004) have reported extensively about this type of problem posing by applying what they call the "What-if?" and "What-if-not" strategies in which problem conditions and constrains are changed.

While solving problem is recognized almost universally as an important mathematical, curricular, and pedagogical activity, problem posing is not, as evidenced by research examining opportunities to pose problems afforded by textbooks (Cai & Jiang, 2016; Cai, Jiang, Hwang, Nie, & Hu, 2016).

Methods of Inquiry

As stated by Pinnegar (1998), self-study is a "methodology for studying professional practice settings" (p. 33). LaBoskey (2004) adds that 'the aim for teacher educators engaged in self-study is to better understand, facilitate, and articulate the teaching-learning process" (p. 857). To illuminate the process of learning to pose mathematical problems, I decided to conduct a self-study research of how I came to learn to pose mathematical problems within dynamic geometry environments.

My Background

I was a high school mathematics teacher for 7 years at a state University in Mexico. After completing a bachelor's degree in Mathematics with a minor in mathematics teaching, I came to the USA and completed a Master's degree and a Ph. D degree in mathematics education. I have about 24 years of teaching experience at the University level. Currently, I teach content and methods courses at the undergraduate and graduate levels, mostly for prospective and practicing teachers.

First encounter with the concept of mathematical problem as the essence of mathematics. As undergraduate, I did not realize the importance of problems for mathematics. I conceived mathematics mainly as a well-integrated body of knowledge involving concepts and procedures connected through theorems whose proofs revealed explicitly the connections. As part of an assignment in one on my methods courses, I read Halmos's (1980) article *The Heart of Mathematics* where he argues that "the heart of mathematics consists of problems". Halmos concludes his article with a call to all instructors that they should "train our students to be better problem-posers" (p. 524). However, I did not interiorize nor appreciate the importance of the idea of learning how to pose problems.

First explicit encounter with the concept of posing problems. As a graduate student, I was one day perusing some books at the library when I encountered by chance Brown & Walter's (1983) *The art of problem posing*. The title of the book intrigued and intimidated me. It intrigued me because it seemed like a book from which I could learn how to pose problems. It intimidated me because learning how to pose problems seemed more like an art, and I did not see myself as a creative person. I left the book where it was and I did not think for a longtime of learning how to enhance my abilities to pose mathematical problems.

First experience on posing problems within dynamic geometric environments. The first problem-posing experience within dynamic geometry environments that I had was with the following problem: Prove that the angle bisectors of the angles of a parallelogram form a rectangle (Landaverde, 1970, p. 85). As a result of this experience and other experiences posing problems without the use of technology, I developed the problem-posing framework displayed in Figure 1 (Contreras & Martínez-Cruz, 2003). Notice that the base problem is the initial given problem whose attributes are to be modified to pose new related problems.

The base problem. I used as base problem the well-known Varignon problem. Typically, the Varignon problem is stated as a theorem (The midpoints of a quadrilateral are the vertices of a parallelogram). I consider this theorem as a mathematical situation within an implicit problem that we can reformulate as a proof problem or as a more open-ended problem. My version of Varignon' problem is as follows: Let E, F, G, and H be the midpoints of the consecutive sides of a parallelogram ABCD. What type of quadrilateral is EFGH?

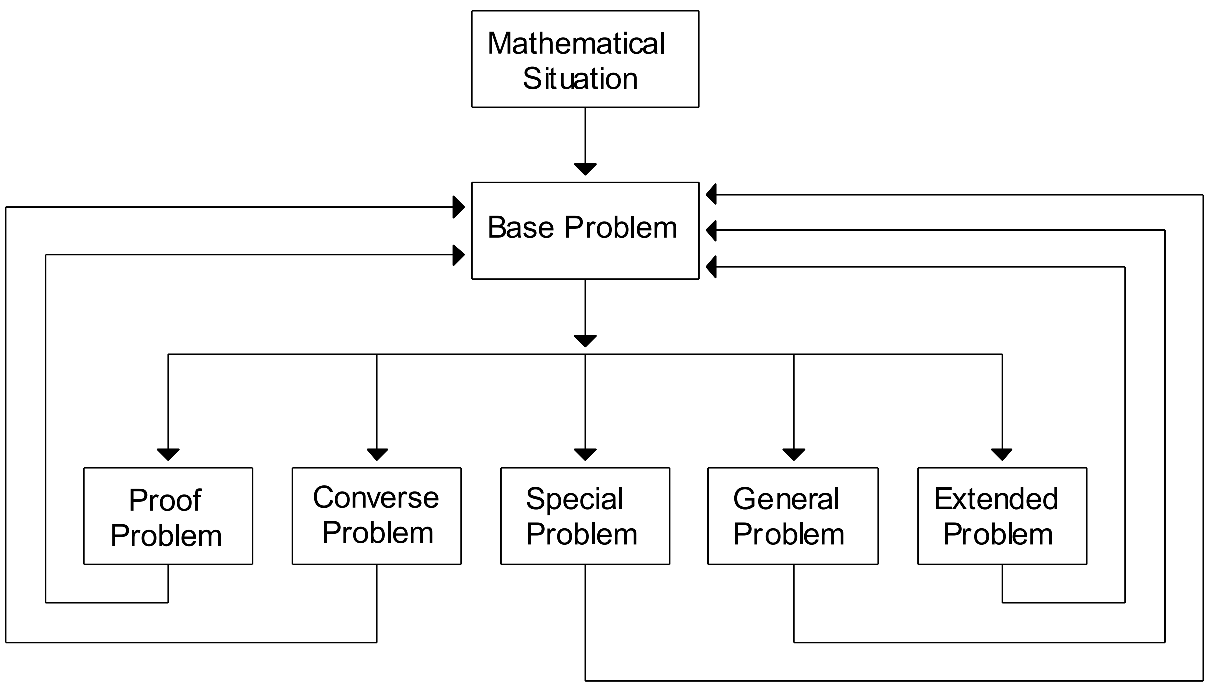

Figure 1: A Problem-Posing Framework

Analysis and Results

Using the problem-posing framework, I posed a diversity of problems that after analysis I classified as special problems, converse problems, extended problems, prove problems, and further extended problems. Typical problems of each of these types are displayed in Table 1.

Table 1: Examples of problems generated using the problem-posing framework

Type of problem	Problem
Special and proof problem	If E, F, G, and H are the midpoints of the consecutive sides of a rhombus ABCD, prove that EFGH is a rectangle.

Converse problem of a special problem	E, F, G, and H are the midpoints of the consecutive sides of a quadrilateral ABCD. If EFGH is a rectangle, what type of quadrilateral is ABCD?
Converse problem of a general problem	If E, F, G, and H are the midpoints of the consecutive sides of a quadrilateral ABCD. If EFGH is a parallelogram, what sort of quadrilateral is EFGH?
Extended problem	ABC is a triangle. Characterize quadrilateral BDEF where D, E, and F are the midpoints of the sides BC, CA, and AB, respectively. (Extended problem to a triangle, which is a degenerate case of a quadrilateral)
Extended and proof problem	Prove that the medial quadrilateral of a kite is a rectangle.
Further extended and proof problem	Prove that the points of intersection of the angle bisectors of the consecutive interior angles of a parallelogram ABCD are the vertices of a rectangle.
Further extended problem	I, J, K, and L are the points of intersection of the sides of a parallelogram ABCD with the interior angle bisectors. What sort of quadrilateral is IJKL?

Conclusion

Researchers (e.g., Crespo, 2003; Crespo & Sinclair, 2008; Nicol, 1999; Silver at al., 1996) report that students, teachers, and prospective teachers typically generate problems that are "predictable, undemanding, ill-formulated, and unsolvable" (Crespo & Sinclair, 2008). While there is some degree of predictability on the types of problems suggested by the problem-posing framework, I used a diversity of language to make them more interesting. I believe that I created a diversity of well-posed problems, each of which is a good and interesting problem because each one opens the mathematics involved or required by the problem (Crespo & Sinclair, 2008). In addition, I used mathematical reasoning and conceptual understanding to generate each problem. The plausibility of each problem was supported with GSP, but I went beyond exploring each problem with GSP and I provide a mathematical solution. In summary, I was actively engaged in the authentic process of doing mathematics. I have made public my second experience in posing mathematical problems within dynamic geometry environments to challenge other mathematics educators to test the problem-posing framework in other appropriate mathematical contexts.

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