A RESEARCH APPROACH ON THE ROLE OF SPACE IN THE CONSTRUCTION OF CONIC SECTIONS

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The results of a literature review of an ongoing research about the construction of the solid conic section and its transition to plane conic section are presented. The review was done in Mathematics Education and History of Geometry Teaching with a main emphasis on the construction of the solid conic as a cutting of the cone, these treatments of conics absent in the curriculum can give meaning to the current treatments of school mathematics. The review concludes with five essential elements that will define the type of study, the study object, and the position of our research in the field.

Keywords: Geometry and Geometrical and Spatial Thinking, High School Education.

  The initial research object for the development of the literature review was *construction of the conic as a cone's cutting and its transition to the plane*. As a first point, one of the problems reported by Mathematics Education research is the absence of conic section's geometric treatments in High School Education, because the conic section's algebraic treatments is in Analytical Geometry class (Pérez-Moguel, 2018; Salinas & Pulido, 2017; Contreras, Contreras & García, 2003); nevertheless, the meanings of these notions are linked to the Plane and Space Geometry.

  In this way, History of Geometry Teaching recognizes: school mathematics are based on the mathematics of the 17th century (Barbin, 2008; 2012; Dennis, 2009); in particular, the solid conic section's construction isn't since 1905 in the curriculum (Barbin, 2012; 2008), and it's replaced by a narrative about the cuttings of a cone made by Apollonius of Perga (Fried, 2007; 2001), and then to define them on the plane from the foci and directrix (as the case may be), without any link to the Apollonius' cuttings (Salinas & Pulido, 2017). Indeed a second point, we identify conics as "a perennial notion with many properties, many theories and contexts, geometric and algebraic approaches, relations between plane geometry and space" (Barbin, 2008, p. 157); therefore, we synthesize the chronology of the study and development of this notion: *Solid Conic Section; Plane Conic Section; Analytical Conic Section* (Coolidge, 1968; Bartolini Bussi, 2005; Bongiovanni, 2007).

  As a third and more important point, the research of Pérez-Moguel (2018) will be a fundamental antecedent because her historical-epistemological study of parabola's geometrical construction as a cone's section, she identifies a series of *actions* and *activities* that encourage us to ask ourselves about the practices associated to construction of the solid conic section and its passage to plane conic section. Among these actions, Pérez-Moguel (2018) highlights the transition between 3D and 2D dimensions in solid parabola's construction, coinciding with Salinas and Pulido (2017) who consider that spatial ability is fundamental in the construction of solid conic sections. Therefore the new object of investigation, modified after the review will be: *spatial processes, and practices associated to solid conic section's geometrical construction relative to the cone's cutting, and its transition to the plane conic section,* in the original text *Apollonius of Perga: Conics*.

References

Barbin, E. (2008). Perennial notions and their teaching. In E. Barbin, N. Stehlíková & C. Tzanakis (Eds.), *History and Epistemology in Mathematics Education: Proceedings of the fifth European Summer University* (pp. 157-161). Pilzen: Vydavatelský servis.

Barbin, E. (2012). Teaching of conics in 19th and 20th centuries in France: on the conditions of changing (1854–1997). In K. Bjarnadóttir, F. Furinghetti, J. Matos & G. Schubring (Eds.), *Proceedings of the second International on the History of Mathematics Education* (pp. 44-59). Lisbon: Universidade Nova.

Bartolini Bussi, M. (2005). The meaning of conics: historical and didactical dimensions. In J. Kilpatrick, C. Hoyles, O. Skovsmose & P. Valero. (Eds.), *Meaning in Mathematics Education* (pp. 39-60). New York, United States: Springer.

Bongiovanni, V. (2007). Étude historique des premieres caracterisations des coniques. *Revista Brasileira de Hitória da Matemática*, 7(14), 439-462.

Coolidge, J. (1968). *A history of the conic sections and quadric surfaces.* New York, United States: Dover Publications, Inc.

Contreras, A., Contreras, M. y García, M. (2003). Sobre la geometría sintética y analítica. La elipse y sus construcciones. *Revista Latinoamericana de Investigación en Matemática Educativa*, 5(2), 111-132.

Fried, M. (2001). Can Mathematics Education and History of Mathematics Coexist? *Science & Education*, 10(4), 391-408. https://doi.org/10.1023/A:1011205014608

Fried, M. (2007). Didactics and history of mathematics: knowledge and self-knowledge. *Educational Studies in Mathematics*, 66, 203-223. https://doi.org/10.1007/s10649-006-9025-5

Pérez-Moguel, Z. (2018). *Una problematización de la parábola en su construcción geométrica* (Tesis de maestría no publicada). Cinvestav-IPN, Ciudad de México.

Salinas, P. & Pulido, R. (2017). Understanding the Conics through Augmented Reality. *EURASIA Journal of Mathematics Science and Technology Education,* 13(2), 341-354.