NAVIGATING COMPLEXITIES IN DEFINITIONS OF LENGTH AND AREA

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Deficiencies in elementary students' conceptual understanding of spatial measurement have persisted, emerging through educational research (e.g., Kamii & Kysh, 2006) and national assessments (e.g., National Assessment of Educational Progress [NAEP]). Investigating several decades of results from the NAEP, Kloosterman, Rutledge, and Kenney (2009) described persistent measurement deficiencies. Research suggests that elementary students struggle with conceptual understanding of spatial measurement (i.e., length, area, volume) and graduating preservice teachers (PSTs) often share their struggles. For example, elementary students struggle in understanding distinctions between area and perimeter and relationships between their measures (e.g., Bamberger & Oberdorf, 2010; Barrett & Clements, 2003; Woodward & Byrd, 1983). The intuitive expectation that measures of perimeter and area always increase or decrease together is an enduring, commonly held misconception (e.g., Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998; Tan Sisman & Aksu, 2016). PSTs, soon to be teaching such concepts, have shown similar misconceptions (e.g., Ma, 1999; Livy, Muir, & Maher, 2012; Wanner, 2019).

We examined definitions related to length and area measurement in 11 textbooks specifically developed for use with preservice elementary teachers in mathematics content courses. Our selection of the textbooks was guided by Raven (2006) and represents a wide range of textbooks that vary in organization, coverage of topics, and attention to pedagogy. The books are written by mathematicians, mathematics educators, or both.

Two researchers adapted and clarified an existing framework to code definitions of spatial measurement in elementary curricula with respect to selected aspects (Gilbertson, He, Satyam, Smith, & Stehr, 2016). We identify the coding unit, a definition, as a focused description of meaning, set apart from other text. We captured definitions of length and area using the textbook index and scanning relevant sections. Two researchers independently coded each definition and met to compare coding and resolve discrepancies.

Based on Stehr and He (2019), we used a four-step measurement process: (1) select an object and measurable attribute, (2) select a unit of measure, (3) compare the attribute of the object with the unit, and (4) express the measure. We provide our analytical frameworks and findings in the poster. In the first step of the measurement process, select an object and an attribute of that object to be measured. A measurable spatial attribute is a characteristic of an object that can be quantified, has dimensionality, takes up space, and often has clear boundaries. To select a unit of measure in the second step, note that the unit could be standard or nonstandard, a reproducible unit that tessellates space, using parts of a unit as needed, and may be be continuous or discrete. In the third and fourth steps, the measure of an attribute is expressed by comparing the attribute to the unit to determine the number of units and parts of units that cover or fill the space without leaving gaps or overlaps. The comparison may include procedural tool use. The final measure of an attribute is expressed as a multiple of the standard or nonstandard unit.

The goal in analyzing textbook definitions and finding variation is not necessarily to point out gaps or failings, because textbooks may add to definitions through tasks or other text. We focus attention on the ways definitions could be written at multiple levels of sophistication and with careful choice of aspects, hoping to open a larger discussion.

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

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