"A SQUARE IS NOT LONG ENOUGH TO BE A RECTANGLE": EXPLORING PROSPECTIVE ELEMENTARY TEACHERS' CONCEPTIONS OF QUADRILATERALS

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Developing knowledge of the characteristics and properties of two- and three- dimensional geometric shapes are considered key content for elementary-aged students to learn. Quadrilaterals, one of the conceptually complex topics in geometry, have a range of attributes such as side length, angle measure, relationships between opposite/adjacent angles/sides, length and bisection of diagonals, etc. Previous research corroborated that transitioning from the introductory information about attributes of these shapes to mentally manipulating and proving the properties is challenging. Many teachers across elementary through senior grades, struggle with geometry content, and consequently, so do their students (Bhagat & Chang, 2015). Research reports that prospective teachers have difficulty in identifying specific shape types based on properties, rather than visual recognition (Burger & Shaughnessy, 1986) and struggle to determine other properties or define the relationships between various quadrilaterals. Drawing on Tall and Vinner's (1981) description of concept-image and concept definition, this study aims to add to the existing literature by answering the research question: *What conceptions do prospective elementary mathematics teachers have of the attributes of the family of quadrilaterals?*

The participants in this study were 19 prospective elementary teachers (2 male and 17 female) between 17-24 years of age enrolled in a content course designed to support knowledge of geometry and measurement in the elementary curriculum using a problem-solving approach. Data sources include the PSTs responses on a 26-item (15 closed and 11 open-ended) geometry tests. Findings showed that PSTs reasonings seemed to be fixed towards certain images rather than mental manipulation of the attributes and concept definitions (Vinner, 1991). For instance, PSTs considered rectangle as a figure with two long parallel sides and two shorter parallel sides and four 90° angles, while defined parallelogram as a slanted shape. PSTs tended to over-generalize the attributes of certain common quadrilaterals showing a continuous exposure to a geometrical shape in specific orientation (e.g., square having a horizontal base). While responding to the question, 'do the diagonals of kite bisect each other?', most of the PSTs responded incorrectly having generalized the properties of the diagonals of a square, rectangle, and rhombus.

The results of this study highlight the importance of emphasizing the two-way interaction between concept image and concept definition by explicitly showing interactions between theoretical and practical application of the learned ideas. Creating the definitions of quadrilaterals based on understanding of attributes (Fujita & Jones, 2007; Usiskin & Griffin, 2008), and exploring the inclusive relationship between various quadrilaterals (Fujita, 2012) may support PSTs in developing conceptions using non-prototypical examples. In this regard, time, high-quality tasks, and appropriate scaffolding are essential to strengthening understanding of quadrilaterals specifically, and geometric concepts more broadly.

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