

COMPREHENSION OF COMPLEX MATHEMATICAL TASKS WITHIN THE SCHOLARIZED CULTURAL ENVIRONMENT IN THIRD-GRADE

ENTENDIMIENTO DE TAREAS MATEMÁTICAS COMPLEJAS DENTRO DEL ENTORNO CULTURAL ESCOLARIZADO EN TERCER GRADO DE PRIMARIA

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The understanding of complex quantitative relationships requires analyzing the cognitive processes of mathematical representations in students (Vygotsky, 1988), within school cultural environments. The development of concepts at an early age represents an opportunity in the forming of habits of abstract thinking for students of basic education (Carpenter et al, 2005; Carraher & Schliemann, 2007). The understanding of the quantitative relationships of mathematical concepts requires several areas: 1) the analysis of cognitive processes, through their mathematical representations and their discourse; 2) contemplate the school cultural environment in which children learn 3) the possibility of understanding complex concepts such as the order of operations or algebraic thinking.

For the present study, the understanding of quantitative relationships of 30 third-grade primary school students was analyzed, in complex mathematical tasks of the order of operations and algebraic thinking, in their schooled cultural environment (public primary). A teaching experiment was implemented based on tests of mathematical competence, concrete manipulative tasks based on part-whole relation (Davydov, 1962), and a content analysis of the students' discourse. The assessment of understanding was based on reactive tests of the order of operations and algebraic thinking. These tests were complementary with interviews with each participant. The tasks in the three stages of the teaching experiment were correlated with each other (RhO Spearman) to assess the internal consistency of the assessment.

The results indicate that 27% of the third-grade students expressed at least a potential understanding of the algebraic thinking tasks. The meaning of the variable and the unknown was linked to unknown or hidden quantities. 47% also expressed this understanding of the order of operations tasks. The justification for using the order of operations was the union between quantities that are multiplied compared to quantities that are added. The average scores of the children during the 10 sessions of the sequence had a high correlation with the average scores of the students in the tasks of algebraic thinking (0.733 ($p < 0.000$)) and the order of operations (0.769 ($p < 0.000$)). The results are compatible with the findings of multiple investigations of the order of operations or PEMDAS (Glidden, 2008; Gunnarsson et al., 2016; Linchevski & Livneh, 1999; Papadopoulos & Gunnarsson, 2020; Taff, 2017; Zorin & Carver, 2015).

It is possible, thanks to the potential of the formation of complex thinking habits, to understand algebraic thinking tasks and the order of operations schoolchildren at an early age, from concrete and significant experiences; When a child comes to understand and see the significance of mathematical tasks, he becomes enthusiastic about learning mathematics.

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