LEARNING THROUGH ACTIVITY (LTA) IN SUPERIOR EDUCATION: THE CASE OF THE HEINE-BOREL THEOREM

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The research program called Learning Through Activity (LTA) had its origins in Simon (1995). This research program has the objective of creating an integrated theory of conceptual learning, and the design of instructional mathematical activities through the use of Hypothetical Learning Trajectories (HLTs). Thus, this HLTs play a crucial role in the LTA program. This theoretical framework implements the constructivist theory of education, developed by Piaget (1970), as well as its applications in mathematics pedagogy by von Glasersfeld (1995).

In the last decades, the community of mathematics educators has become very interested in continuing to expand this theoretical framework, in an effort to incorporate social, cultural and psychological theories in the teaching-learning processes involved in a classroom context.

During this years, according to Stylianides & Stylianides (2009, 2018), research about teaching through this kind of activities has shown very promising results in the basic levels of education, and this being the case, HLTs as part of a mathematical teaching cycle have become one of the main referents about how to design activities to guide students learning according to the constructivist theory of education, for example (Leikin & Dinur, 2003; Simon, Kara, Placa & Avitzur, 2018; Stylianides & Stylianides, 2018). However, although HLTs seem to be a very promising way to approach mathematics pedagogy, there has been little research about their implementation in the higher levels of mathematical education (Simon et al, 2018).

In this Poster, we present a synthesis of a didactical proposal, based on the LTA program's approach, that includes a main HLT with the goal of guiding the student towards a proof of the Heine-Borel theorem, and other auxiliary HLTs, that will provide the student with the necessary tools to prove the theorem. We do this with the intent to investigate the efficiency of the LTA framework in the higher levels of mathematical education.

The proposed trajectories begin with the definition of open covers, and continue through some supporting theorems, such as the theorem that guarantees the compactness of closed subsets of compact sets, and the theorem that guarantees the compactness of any K-cell, before culminating in the proof of the Heine-Borel theorem, which states that any subset A, of the Euclidean space Rn, is compact if and only if A is closed and bounded.

The study is addressed to university students beginning their studies in the topology of Rn in the Faculty of Sciences of the "Universidad Nacional Autónoma de México" (UNAM), the most important public University of México. We based the mathematical part in Rudin (1986), and other classical texts such as Bartle (2011).

Lastly, this poster will also include a synthesis of a metric based on Toulmin's argumentative model that will be used to evaluate the knowledge acquired by the students through the learning trajectories. In the last years, this model has been used by several researchers in mathematical education to analyze non formal arguments, for example (Pedemonte, 2007; Simpson, 2015; Zazkis, Weber & Mejia, 2016; Herrera, Rivera & Aguirre, 2019).

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