FUNKY PROTRACTORS CREATED BY PROSPECTIVE TEACHERS

Hamilton L. Hardison
Texas State University
HHardison@txstate.edu

Hwa Young Lee
Texas State University
hylee@txstate.edu

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Angles and angle measure are important and frequently leveraged concepts throughout school mathematics curricula. Yet, relative to other quantities like length, area, and volume, very little scholarly literature addresses how students and teachers understand angle measure (Smith & Barrett, 2017). From the scant extant literature, it is clear that developing productive conceptions of angle measure is non-trivial for students and teachers alike (Akkoc, 2008; Lehrer, Jenkins, & Osana, 1998; Smith & Barrett, 2017). In the U.S., individuals' challenges in quantifying angularity may be partially attributed to instructional approaches that (a) overemphasize the use of conventional protractors to measure angles and (b) fail to address how the design of these conventional tools renders them appropriate for measuring angles (Moore, 2012). This is especially problematic given that well-prepared beginning teachers must be skilled in explaining how to select appropriate tools for particular mathematical goals (Association of Mathematics Teacher Educators, 2017).

To occasion conversation and reflection about angular measurement and protractors in our geometry courses for prospective teachers, we designed tasks involving a collection of non-standard tools that might be used to measure angles. We refer to these tools as *funky protractors* (Hardison & Lee, 2020a). For each funky protractor we designed, we altered one or more features to differentiate it from a conventional protractor (e.g., uncommon shape, equally spaced linear or angular intervals, non-standard angular unit of measure, etc.). We intentionally designed some funky protractors to be valid tools for measuring angles and others to be invalid; in previous implementations, we have asked prospective teachers to determine which funky protractors are valid tools for measuring angles and to justify their decisions. Thus, funky protractor tasks are the angular analogue of the "strange ruler" tasks others have used to promote critical thinking about linear measure (Dietiker, Gonulates, & Smith, 2011). Elsewhere, we have discussed prospective teachers' decisions regarding the validity of funky protractors, as well as the strategies they leveraged to support their decisions (Hardison & Lee, 2020b, this volume).

In this poster presentation, we report on an extension of the funky protractor tasks, which we implemented with prospective middle and secondary teachers enrolled in one section of a geometry content course at a large public university. After evaluating the validity of four funky protractors and engaging in a whole-class discussion, prospective teachers were asked to create two of their own funky protractors: one that would be a valid tool for measuring angles and one that would not be a valid tool for measuring angles. We present examples of the funky protractors that prospective teachers created and analyses of these items. In particular, we (a) summarize how successful prospective teachers were in creating valid and invalid tools for angular measurement, (b) describe the features prospective teachers manipulated when designing their own funky protractors, and (c) discuss prospective teachers' perspectives on the pedagogical utility of funky protractors.

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