ADVANCING REASONING AND PROOF IN SECONDARY MATHEMATICS CLASSROOMS: INSTRUCTIONAL MODULES FOR SUPPORTING TEACHERS

Orly Buchbinder
University of New Hampshire
orly.buchbinder@unh.edu

Sharon McCrone
University of New Hampshire
Sharon.McCrone@unh.edu

Keywords: Reasoning and Proof, Instructional activities and practices

Bringing to life a vision of mathematics teaching that emphasizes reasoning and proof across topics and grade levels can be a challenging task for teachers. To support and enhance teachers' content and pedagogical knowledge for teaching reasoning and proof, we developed and systematically studied a capstone course for preservice teachers (PSTs) *Mathematical Reasoning and Proving for Secondary Teachers*. The course emphasized integration of reasoning and proof with teaching secondary mathematics across *any* curricular topic, not just high-school geometry.

During our design-based research project, we developed and tested four instructional modules, each focusing on a particular proof theme: (1) direct reasoning and argument evaluation, (2) conditional statements, (3) quantification and the role of examples in proving, and (4) indirect reasoning. Each module comprised three types of activities: *crystalize, connect* and *apply*. The *crystalize* activities aim to help PSTs refresh their memory of a particular proof theme, within secondary school content. The PSTs enhance their knowledge by solving problems, as well as discussing and clarifying questions or misconceptions, e.g., the difference between proof by contradiction and by contrapositive. In the *connect* activities, the PSTs have an opportunity to connect their mathematical knowledge with knowledge of students' conceptions. The PSTs read cases or watch video or animations of classroom scenarios depicting students working on problems in a particular proof theme. For example, in the *connect* activity of the Quantification and the Role of Examples in Proving module the PSTs analyze a dialog between three students contemplating how to interpret a statement "There exist three consecutive even numbers whose sum is a multiple of four" and what kinds of examples can prove or disprove (if possible) this statement. Next, the PSTs envision possible pedagogical moves to support student thinking in the scenario. The *apply* tasks invite PSTs to identify opportunities in secondary curriculum to make proof themes explicit to students, and to develop a lesson plan that achieves that goal. In our course, we had PSTs implement these lessons in actual middle and high school classrooms, videotape themselves, and reflect of their teaching (Buchbinder & McCrone, 2018).

All four course modules were designed to be independent from each other and applicable for individual use in courses for PSTs or teacher workshops. The design of the instructional modules is grounded in the literature on best practices for teacher learning and professional development, such as focusing on deepening both content and pedagogical knowledge, engaging teachers in active learning experiences, and making direct connections to teachers' classroom practices (American Federation of Teachers 2002; Boston & Smith, 2009, 2011; Copur-Gencturk & Papakonstantinou, 2015). In our poster, we will show the four modules, explicate design features underlying their development, and provide evidence for the effectiveness of the modules.

Acknowledgments

This research was supported by the National Science Foundation, Award No. 1711163. The opinions expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation.

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

References

American Federation of Teachers. (2002). *Principles for professional development: AFT's guidelines for creating professional development programs that make a difference.* Washington, DC: Author.

Boston, M. D., & Smith, M. S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40*(2), 119–156.

Boston, M. D., & Smith, M. S. (2011). A 'task-centric approach' to professional development: Enhancing and sustaining mathematics teachers' ability to implement cognitively challenging mathematical tasks. *ZDM, 43*(6–7), 965–977.

Buchbinder, O. & McCrone, S. (2018) Mathematical Reasoning and Proving for Prospective Secondary Teachers. *Proceedings of the 21st Annual Conference of the Research in Undergraduate Mathematics Education, Special Interest Group of the Mathematical Association of America* (pp. 115–128). San Diego, CA.

Copur-Gencturk, Y. Olanoff & Papakonstantinou, A. (2015). Sustainable changes in teacher practices: A longitudinal analysis of the classroom practices of high school mathematics teachers. *Journal of Mathematics Teacher Education*. DOI 10.1007/s10857-015-9310-2.