PROBLEM POSING IN PARTITIVE AND QUOTITIVE DIVISION

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Problem posing (Brown & Walter, 2005) can improve problem-solving (e.g., English, 1998), expand mathematical creativity (e.g, Voica & Singer, 2012), and provide instructors with valuable insight into student understanding (e.g, Silver, 1994). Problem posing can also give students ownership in learning and provide students with authentic modelling processes (Hanson & Hana, 2015). We aim to investigate problem posing with partitive and quotitive models of division. In quotitive division, the divisor is the number in each group. In partitive, the divisor is the number of groups (Neuman, 1999). Asking students to make the distinction between the two models of division has been shown to help students better understand place value (Bicknell, Young-Loveridge & Simpson, 2017), division with decimals (Okazaki & Koyama, 2005), division with remainder (Lamberg & Wiest, 2012), and the division algorithm (Silver, 1987).

Methods

In a 100-level math course at a private liberal arts university, which incorporated various problem posing tasks, students (n=38) were given the task "For 14÷2=7, write (a) one word problem that demonstrates a partitive model and (b) one that demonstrates a quotitive model." Students were then asked to create children's books that contained both models of division, to be donated to the Girls & Boys Club as a local outreach project. We seek to address: how do students respond when asked to pose problems demonstrating the two models of division?

Results

In the written task, 14 (36.8%) correctly demonstrated both models of division. We observed gaps in understanding and nuanced misconceptions, which inform teacher education. For example, students commonly provided partitive when asked for a quotitive model, provided problems not solved by 14÷2=7, and struggled with the concept that word problems from both partitive and quotitive models can correspond to the same equation.

Table 1: Summary of Student Response Themes

	Incorrect Model	Used Own Numbers	Rearranged Given Equation	Phrased as Statement
Part (a) [Partitive]	4 (10.5%)	5 (13.6%)	1 (2.6%)	15 (39.5%)
Part (b) [Quotitive]	14 (36.8%)	6 (15.8%)	10 (26.3%)	16 (42.5%)

Furthermore, we observed that all (14 of 14) students who drew an image with their response on part (a) provided a correct model. The analysis of the children's book task, which revealed what contexts the students chose, yielded themes of real-world relevance, fairness, and novelty. Future research includes exploring the impact of problem posing on problem solving in division and student perceptions of a course that utilizes problem posing tasks.

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