PROMPTING QUANTITATIVE REASONING PATTERNS WITH MATHEMATICAL MODELING PROBLEMS

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Contributing to the call for improving secondary instruction, the National Council of Teachers of Mathematics (NCTM, 2000) emphasized that students should gain quantitative reasoning abilities that lead to work with quantitative information and be able to use formulas and graphs to represent how quantities in real-life phenomena are related to one another and change together. Ellis (2007) suggested that one way to support students' quantitative reasoning is by engaging them in problem-solving activities that require (a) exploring how changing initial quantities will affect the emergent quantities, (b) determining how to adjust the initial quantities while keeping the emergent quantities constant, and (c) determining how to adjust the emergent quantities with the initial quantities (p. 475). In that sense, mathematical modeling problems naturally provide an environment for fostering and nurturing quantitative reasoning skills (Carlson, Larsen, & Lesh, 2003; Thompson, 2011).

In this poster presentation, a partial report of a larger study, we examined two tenth-graders'—Carlos and Ahmad (pseudonyms)—quantitative reasoning patterns and quantification processes while they solved mathematical modeling problems. Each student was interviewed one-on-one and given four modeling problems. Each interview was approximately 60 minutes long, and the students were encouraged to explain their reasoning processes (Ericsson & Simon, 1998). In the data analysis, we adopted the quantitative reasoning in context (QRC) framework (Mayes, Peterson, & Bonilla, 2013), which has four elements: (a) the quantification act (QA), the ability to identify the mathematical objects and their unit measures; (b) quantitative literacy (QL), the ability to identify, compare, manipulate, and draw conclusions from variables; (c) quantitative interpretation (QI), the ability to discover patterns and trends; and (d) quantitative modeling (QM), the ability to create representations to explain the problem and to revise them based on their fit into reality (p. 130).

The initial findings indicate that both the students were comfortable when identifying variables and their unit measures. Both recognized that they had assigned numbers as assumptions. Two distinct patterns emerged when comparing and manipulating the unit measures throughout the four modeling problems: (a) when Carlos assigned numbers, he primarily used the smallest unit as a measure and made calculations from the part to the whole *(inductive thinking approach)* (Simon,1996), whereas Ahmad always simplified the whole unit to reach the smallest unit at the end and made calculations from the whole to the part (*deductive thinking approach*) (Simon, 1996), (b) those thinking approaches impacted their quantitative interpretations on the mathematical models they had created (i.e., tables and graphs). For example, while Carlos explained the patterns on his graphs in a *descriptive modeling way* (Maaß, 2010) as explaining or forecasting the real-life situation, Ahmad's explanations were solely focused on the generalized mathematical results and mathematical accuracy in a *normative modeling way* (Maaß, 2010). In the presentation, the excerpts will be shared under each reasoning pattern, and possible instructional implications will be discussed.

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