THE ADVANCED TACITLY NESTED NUMBER SEQUENCE: WHY DOES IT MATTER IN THE MIDDLE GRADES?

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Steffe and Cobb (1988) defined four stages of elementary students' understanding of number, which are based on the levels of units that students construct and coordinate (Ulrich, 2015, 2016a). First is the initial number sequence, marked by counting on; next is the tacitly nested number sequence (TNS), marked by double counting and constructing composite units (i.e., units of unit) in activity; third is the explicitly nested number sequence, which is defined by multiplicative reasoning; and finally, the generalized number sequence. In her work with sixth-grade students, however, Ulrich (2016b) defined an additional stage, the advanced tacitly nested number sequence (aTNS). The name is literal in its meaning: aTNS students can apply the operations of their TNS in advanced ways. Research has since identified substantial numbers of aTNS students in the middle grades (Ulrich & Wilkins, 2017; Zwanch & Wilkins, in review). This study builds on Ulrich's (2016b) definition of an aTNS by characterizing the ways in which aTNS students' numerical and algebraic reasoning typify an *advanced* TNS.

The data for this study were collected from students in grades six through nine at a small school district in the southeastern United States. 18 students (2 TNS, 8 aTNS, 6 ENS, 2 GNS) participated in two, 45-minute, semi-structured clinical interviews. Each student's number sequence attribution was confirmed, and the students' algebraic reasoning was characterized.

One characteristic use of number that distinguished aTNS students was their use of skip counting to solve multiplicative problems. TNS students construct composite units in activity (Ulrich, 2015), thus when asked to solve the bar task (Figure 1), they tended to partition the larger bar into four pieces and add 8 four times. Constructing composites in activity supported double counting, but not multiplicative reasoning. aTNS students operate on composite units (Ulrich, 2016b) making it possible for them to think about repeating composite units of eight. Accordingly, aTNS students tended to partition the larger bar into four pieces, then skip count by 8 four times. Most aTNS students stated that they had multiplied eight times four, but their behavior was inconsistent with multiplication. This shows a retrospective awareness of the multiplicative relation, but even after solving several similar tasks, aTNS students did not generalize the multiplicative nature of one bar task to the others. Algebraically, similar behaviors manifested in students' solutions to solving systems of equations. aTNS students used guess and check methods wherein the checking was heavily dependent on students' skip counting. Thus, aTNS students' operations on composite units supported their solutions to algebraic tasks in a manner that confirms their categorization of having constructed an *advanced* TNS. This also demonstrates how aTNS students' numerical and algebraic reasoning is distinct from their peers, making it necessary to consider how we support their learning in the middle grades.

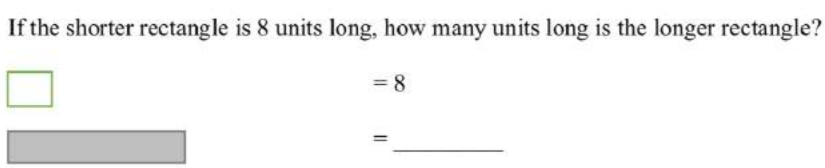

Figure 1. Bar Task from Ulrich and Wilkins (2017)

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