ELEMENTARY ALGEBRAIC THINKING WITH PATTERNS IN TWO VARIABLES

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Usiskin (1999) described four conceptions of algebra: Algebra as Generalized Arithmetic, Algebra as a Study of Procedures for Solving Certain Kinds of Problems, Algebra as the Study of Relationships among Quantities, and Algebra as the Study of Structures. The Algebra as the Study of Relationships among Quantities conception relates to the NCTM (2000) Algebra Standard expectation that students "understand patterns, relations, and functions" (p. 37). Algebraic thinking "includes being able to think about functions and how they work, and to think about the impact that a system's structure has on calculations" (Driscoll, 1999, p. 1). Analyzing students' algebraic thinking with patterning tasks in two variables allows researchers to understand how students think about functions, how they work, and how the representation provided in the question impacts student thinking about the structure of the problem. In this study, one elementary student solved patterning problems in two variables with different representations during a task-based interview (Goldin, 2000).

Preliminary findings suggest that this student used different reasoning strategies when given pattern problems in two different representations. On a task consisting of a visual pattern of figures growing in an arithmetic sequence, the student visualized how the growth occurred in each successive figure. The student used the rate of growth to compute the size of the figure at future iterations. In the context of this task, the evidence suggests that the student was thinking covariationally (Confrey & Smith, 1994) about the relationship between the increase in figure size and increase in figure number.

When presented with a task showing a linear relationship between values in an input-output table of numbers, the student was asked to determine the output value when the input value was 38. Upon receiving this question, the student intensely looked at the problem before stating:

> Oh, I see it now. Okay, so I see if you multiply this by – each number [points at all the numbers in the left input column] by two and add 1, that's the number on this side [points at all the numbers in the right output column]. So take 15 for example. 15 times 2 is 30, plus 1 is 31 and that is in the out. [15 and 31 correspond to each other in the table. 15 being in the input column and 31 being in the output column].

The student used this mapping between the numbers in the input column and the output column to determine 38 corresponds to 77. In this context, the student used a correspondence approach (Confrey & Smith, 1994) to determine the output when the input was 38.

In conclusion, both tasks contained the same structure as linear functions. However, the student thought differently about how the functions "worked" when given a visual pattern of growth as opposed to when given an input-output table. This student showed the capacity to reason through covariation and correspondence while the context of the problem may have influenced the approach. The poster presentation will provide evidence and vignettes from the task-based interview.

References

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