DOMAIN APPROPRIATENESS AND SKEPTICISM IN VIABLE ARGUMENTATION

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Several recent studies have focused on helping students understand the limitations of empirical arguments (e.g., Stylianides, G. J. & Stylianides, A. J., 2009, Brown, 2014). One view is that students use empirical argumentation because they hold empirical proof schemes—they are convinced a general claim is true by checking a few cases (Harel & Sowder, 1998). Some researchers have sought to unseat students' empirical proof schemes by developing students' *skepticism*, their uncertainty about the truth of a general claim in the face of confirming (but not exhaustive) evidence (e.g., Brown, 2014; Stylianides, G. J. & Stylianides, A. J., 2009). With sufficient skepticism, students would seek more secure, non-empirical arguments to convince themselves that a general claim is true. We take a different perspective, seeking to develop students' awareness of *domain appropriateness* (DA), whether the argument type is appropriate to the domain of the claim. In particular, DA entails understanding that an empirical check of a proper subset of cases in a claim's domain does not (i) guarantee the claim is true and does not (ii) provide an argument that is acceptable in the mathematical or classroom community, although checking *all* cases does both (i) and (ii). DA is distinct from skepticism; it is not concerned with students' confidence about the truth of a general claim.

We studied how ten 8th graders developed DA through classroom experiences that were part of a broader project focused on developing viable argumentation. One important classroom task in the project was the Circle-and-Spots problem (Stylianides, G. J. & Stylianides, A. J., 2009, Brown, 2014), which was meant to develop DA and to provide a rationale for why empirical arguments are not considered viable. Semi-structured interviews were conducted, in which we provided students with the claim "For every whole number value of n, if you compute $7n - 1$ you will not get a perfect square," and "Thomas'" empirical argument that checked the first seven cases in the claim's domain. Students were asked questions such as if they thought the claim was true, whether the argument was viable, and what they would have to do to make the argument viable. Thematic analysis was used to develop themes among the student responses (*Braun & Clarke, 2006)*.

After collapsing themes, we found that five of the ten students displayed robust understanding of DA. They said that Thomas' argument was not viable because it did not account for all cases in the claim's domain. All of them suggested both of the following ways to make Thomas' argument viable: (a) restrict the domain to just the seven cases that were checked or (b) find some sort of "equation," "pattern," or "relationship" to show why the claim was always true. Two students nonetheless expressed confidence that the claim was true, supporting our view that DA is distinct from skepticism. Two other students showed partial understanding of DA; the other three displayed empirical reasoning. The results provide evidence of how DA can develop in middle grades, and raise the question of how robustly DA can develop without students having significant prior experience with viable general arguments.

References

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101.

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

Brown, S. A. (2014). On skepticism and its role in the development of proof in the classroom. *Educational Studies in Mathematics*, 86, 311-335. https://doi.org/10.1007/s10649-014-9544-4

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), Research in collegiate mathematics education III (pp. 234-283). Providence, RI: *American Mathematical Society*.

Stylianides, G. J., & Stylianides, A. J. (2009). Facilitating the transition from empirical arguments to proof. *Journal for Research in Mathematics Education*, *40*, 314–352. Retrieved from https://www.nctm.org/publications/journal-for-research-in-mathematics-education/