POSING PROBLEMS ABOUT GEOMETRIC SITUATIONS: A STUDY OF PROSPECTIVE SECONDARY MATHEMATICS TEACHERS

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Mathematics has developed into an extensive body of knowledge because there is, and has been, a continuous search for finding solutions to problems posed by someone. Therefore, problem posing is a fundamental activity of doing mathematics (Brown & Walter, 1983, 1993; Halmos, 1980; Contreras, 2019, 2020; Kilpatrick, 1987; Polya, 1973; Silver, 1994). Even though some researchers (Crespo, 2003; Crespo & Sinclair, 2008; Ellerton, 1986a, 1986b, 1988; English, 1996, 1997, 1998, 2003; Lavy & Shriki, 2010; Silver & Cai, 1996; Silver et al., 1996) have provided some insights about issues pertaining to this line of investigation, we do not know enough about the extent to which preservice teachers, who themselves are students, are able to pose problems by modifying the conditions of a given problem. In fact, the research community continues to investigate the different aspects of teaching and learning how to pose mathematical problems (Ellerton, 2013; Felmer, Pehkonen, & Kilpatrick, 2016; Silver, 2013; Singer, Ellerton, & Cai, 2013, 2015) including analysis of textbooks (Cai & Jiang, 2016; Cai et al., 2016).

From a mathematical point of view, generalizing, proving general statements, and generating problems by considering converse-type problems, are important mathematical activities. Generalizing mathematical patterns is one of the most important processes that contributes to the development of mathematics. According to Sawyer (1982), generalizing "is probably the easiest and most obvious way of enlarging mathematics knowledge" (p. 55). Proving these general statements is one of the central activities of doing mathematics. When creating or discovering a theorem, it is often worthwhile to investigate whether the converse of the theorem holds or what additional conditions or restrictions must be added for having a converse-type theorem.

In this study, 17 prospective secondary mathematics teachers were asked to pose problems related to each of four geometric situations. The four problem situations were chosen as to allow for posing general problems, problems about proving general formulas, and converse-type problems. The students generated a total of 225 responses (199 mathematical problems or questions, 4 nonmathematical problems or questions and 22 statements). The 199 mathematical problem were categorized as well-posed problems (168) and ill-posed problems (31). I used Author's (1998) and Moses, Bjork, & Goldenberg's (1990) frameworks for analyzing the strategies that the students used to pose the problems. The framework includes mainly the following seven strategies to pose the problems: variation of unknowns, variations of knowns or givens, variations of restrictions, reversing knows and unknowns (converse-type problems), generalizing, thinking of patterns, and proving (Contreras, 2003; Moses et al., 1990).

The most common strategies used by the students to pose the problems, and number of problems, were: generalization (38), variation of knowns (25), variation of unknowns (21), and a combination of strategies (12). Even though students generated a diversity of problems, only 10 students posed general problems and only two students posed at least one general problem for each geometric situation. In addition, the students rarely posed converse-type problems and proving problems. Given that most of the students were majoring in mathematics, the findings are not very encouraging. Thus, appears to be a need for prospective mathematics secondary mathematics teachers to learn how to pose these types of problems.

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