HOW TO POSE IT: DEVELOPING A PROBLEM-POSING FRAMEWORK

José N. Contreras
Ball State University
jncontrerasf@bsu.edu

Posing problems is an important mathematical activity. In fact, Halmos (1980) views problems as the essence of mathematics and he, as well as other mathematicians (e.g., Polya, 1973) and mathematics educators (e.g., Brown & Walter, 1983, 1993; Silver 1994), argue that we should prepare our students to become better problem posers. It is not surprising then that professional organizations (e.g., Australian Education Council, 1991; National Council of Teacher of Mathematics [NCTM], 1989, 1991, 2000) have called for increased attention for students to be given opportunities to "create new problems by modifying the conditions of a given problem" (NCTM, 1991, p. 95). The research community still continues to investigate the different aspects of teaching and learning how to pose mathematical problems (Felmer, Pehkonen, & Kilpatrick, 2016; Silver, 2013; Singer, Ellerton, & Cai, 2013, 2015).

To help my students and I to become better problem posers within dynamic geometry environments, I developed a problem-posing framework. The problem-posing framework includes the following systematic strategies to pose new problems by modifying the conditions of a given problem: reversing, proving, specializing, generalizing, extending, and further extending. The problem-posing framework has been a powerful tool that has helped both my students and I to create new problems related to a given problem within dynamic geometry environments. The initial problem from which we created new problems was the following: What type of quadrilateral has as vertices the points of intersection of the angle bisectors of the angles of a parallelogram? I will refer to this problem as the base problem.

During the poster presentation, I will display the problem-posing framework and illustrate its usefulness with some of the problems that I and my students have generated by systematically varying the attributes of the base problem. Examples of posed problems include the following:

Problem 1: The vertices of quadrilateral EFGH are the points of intersection of the angle bisectors of a quadrilateral ABCD. If EFGH is a rectangle, what sort of quadrilateral is ABCD? (Converse or reverse problem)

Problem 2: Let E, F, G, and H be the points of intersection of the angle bisectors of the angles of a rectangle. Prove that EFGH is a square or a point. (Special and proof problem)

Problem 3: What kind of quadrilateral has as vertices the points of intersection of the angle bisectors of the consecutive angles of a quadrilateral? (General problem)

Problem 4: Prove that the angle bisectors of the angles of a kite are concurrent. (Extended and proof problem)

Problem 5: The vertices of quadrilateral EFGH are the points of intersection of the consecutive exterior angles of an isosceles trapezoid ABCD. Characterize EFGH. (Further extended problem)

Problem 6: Let E, F, G, and H be the points of intersection of adjacent angle trisectors of the interior angles of a parallelogram. What type of quadrilateral is EFGH? (Further extended problem)

The author will also provide solutions to problems that are supported by proofs in some cases or conjectures supported by empirical evidence in other cases (such as geometric diagrams created with Dynamic Geometry Software or numerical examples).

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

References

Australian Education Council, Curriculum Corporation (Australia). (1991). *A national statement on mathematics for Australian schools: A joint project of the states, territories and the Commonwealth of Australia initiated by the Australian Education Council.* Carlton, Vic: Curriculum Corporation for the Australian Education Council.

Brown, S., & Walter, M. (1983). *The art of problem posing*. Hillsdale, NJ: Erlbaum.

Brown, S. I., & Walter, M. I. (1993). *Problem posing: Reflections and applications*. Hillsdale, NJ: Erlbaum.

Felmer, P., Pehkonen, E., & Kilpatrick, J. (Eds.). (2016). *Posing and solving mathematical problems: Advances and new perspectives*. New York, NY: Springer.

Halmos, P. R. (1980). The Heart of Mathematics. *The American Mathematical Monthly*, 87(7), 519-524.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: The Author.

National Council of Teachers of Mathematics. (1991). *Standards for teaching mathematics*. Reston, VA: The Author.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: The Author.

Polya, G. (1973). *How to solve It: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics*, 14(1), 19-28.

Silver, E. A. (2013). Problem-posing research in mathematics education: Looking back, looking around, and looking ahead. *Educational Studies in Mathematics*, 55(1), 157-162.

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2013). Problem-posing research in mathematics education: new questions and directions. *Educational Studies in Mathematics*, 83(1), 1-7.

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2015). *Mathematical problem posing: From research to effective practice.* New York, NY: Springer.