

CONCEPTIONS AND CONSEQUENCES OF WHAT WE CALL ARGUMENTATION, JUSTIFICATION, AND PROOF

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Since PME-NA 2015, a working group has explored how argumentation, justification, and proof are conceptualized in the extant mathematics education literature and the consequences of these conceptualizations. We feature work from a forthcoming book in which scholars have critically examined the consequences of using particular conceptions of argumentation, justification, and proof as lenses for examining classroom practice from elementary through tertiary grade levels. Session participants will have opportunities to analyze data sets across grade bands and reflect on chapter authors' analyses of those data sets. Discussants will explore implications regarding how definitional choices impact both research and teaching practice. During the final session, we consider the next phase of the working group's focus for future PME-NA conferences.

Keywords: Reasoning and Proof; Advanced Mathematical Thinking; Teacher Knowledge

Theoretical Backgrounds

Although there is a large and growing body of research in mathematics education focused on argumentation, justification, and proof (Cirillo et al., 2015), the definitions and conceptualizations of these terms differ according to the perspective of the researcher, the focus of the research, and the particular data being analyzed (Reid & Knipping, 2010). Consequently, these differences cause inconsistencies in the literature. For example, there are debates about whether argumentation and proof are deeply intertwined or fundamentally separate activities, and there are inconsistencies related to students' success and difficulties with proof (Stylianides et al., 2017). Unpacking and making sense of these kinds of inconsistencies and debates has been a core focus of the Argumentation, Justification, and Proof working group.

History of the Working Group

The *Conceptions and Consequences of What We Call Argumentation, Justification, and Proof* Working Group (AJP-WG) met for the first time during the 37th Annual Meeting of the North American Chapter of the Psychology of Mathematics Education (PME-NA) in 2015. The group then met for three additional years in 2016, 2017, and 2018. The AJP-WG sessions were well-attended each year, and the group is active in between meetings. For example, following the 2015 and 2016 meetings, AJP-WG members published white papers to disseminate the group's work during the sessions (i.e., Cirillo et al., 2016; Staples et al., 2017). Following the 2018 meeting, AJP-WG members began work on an edited book. Across the authoring teams of the AJP-WG papers, the white papers, and the book, over 40 scholars have been involved in the group's work, including many graduate students. The goal of this colloquium is to share ideas from the forthcoming book (Bieda, Conner, Kosko, & Staples, forthcoming), which have developed during previous PME-NA AJP-WG sessions.

Areas of Discussion and Plan for the Research Colloquium

Each day will feature presentations from two chapter authors who analyzed the same grade level classroom transcript from different perspectives (e.g., justification and proof). Prior to author presentations, participants will have opportunities to review each transcript (i.e., the data set) and

make their own observations. Participants will consider implications of the authors' definitional choices and conclusions with respect to research, working with teachers, or both.

Day 1: Opening Session and Focus on the Elementary Grades

The goals for the first session will be to (1) share the origins and evolution of the AJP-WG that motivated this research colloquium (2) introduce the aims and structure of the edited book that resulted from past efforts of the AJP-WG, and (3) have a set of book authors, whose chapters focused on justification and proof in the elementary grades, share reflections from their analyses. To achieve these goals, we anticipate the following organizational structure: M. Cirillo will lead Activities 1 and 2 [30 minutes] and K. Bieda will facilitate a review of the data set used by elementary section authors in the book [30 minutes]. Then, E. Thanheiser and C. Walkington will co-present major findings from their analyses of the elementary data set [20 minutes]. Finally, we will facilitate audience questions and discussion [10 minutes].

Day 2: Focus on the Middle Grades

The goals for the second day will be to explore the middle grades data and author analyses and to preview the data set for Day 3. K. Bieda will facilitate groups in reviewing the middle grades data set used by the book section authors [30 minutes]. Then, C. Gomez and K. Lesseig will share major findings from their analyses of the middle grades data set [20 minutes]. We will then facilitate audience questions and discussion [10 minutes]. Last, D. Plaxco will facilitate groups in reviewing the data set featured in the tertiary (post-secondary) book chapters [30 minutes].

Day 3: Focus on the High School and Tertiary Grades and Next Steps for Working Group

The goals for the third day will be to explore and reflect on the author analyses at the high school and tertiary levels, reflect on the implications of analyses across the presentations, and brainstorm about future working group foci as suggested by implications of the work presented across the three days. First, M. Cirillo will share findings from the Cirillo and Cox high school synthesis chapter which explores the findings of three authors who analyzed a common transcript through the lenses of argumentation, justification, and proof. Similarly, D. Plaxco will share major findings from analyses of the tertiary data set [30 minutes]. We will facilitate questions and discussion [10 minutes]. Then, A. Ellis will share reflections on the set of presentations featured during Day 1-3 of the colloquium [30 minutes]. We will then facilitate a conversation about potential topics for future PME-NA working groups based on implications from the analyses presented across the three days of colloquium sessions.

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