# FOUR COMPONENT INSTRUCTIONAL DESIGN (4C/ID) MODEL CONFIRMED FOR SECONDARY TERTIARY MATHEMATICS

Carol H. Wade, Ph.D.
State University of New York (SUNY)
Brockport
cwade@brockport.edu

Christian Wilkens, Ed.D.
State University of New York (SUNY)
Brockport
cwilkens@brockport.edu

Gerhard Sonnert, Ph.D.
Science Education Department
Harvard-Smithsonian Center for Astrophysics
gsonnert@cfa.harvard.edu

Philip M. Sadler, Ed.D.
Science Education Department
Harvard-Smithsonian Center for Astrophysics
psadler@cfa.harvard.edu

*Cognitive Load Theory's Four Component Instructional Design (4C/ID) Model has been used in mathematics education but not confirmed as an instructional theory. Using the Factors Influencing College Success in Mathematics (FICSMath) project and confirmatory factor equation modeling, we empirically validated the model and created the 4C/IDMath Model. Instructional experiences of respondents completing the FICSMath survey were mapped to the theoretical components of the 4C/ID Model. The Mathematical Learning Task, Conceptual Understanding, Procedural Fluency, and Practice for Recall Components correspond to the Learning Task, Support, Procedure, and Part Task Components, respectively, from the original 4C/ID Model. The 4C/IDMath Model can be used to guide instruction in secondary precalculus and calculus courses to support transfer of learning to single variable college calculus.*

Keywords: Research Methods, Design Experiments, Secondary-Tertiary Transition in Mathematics

## Theoretical Perspective

Cognitive load theory (CLT) was introduced in the 1980s as an instructional theory based on well accepted aspects of human cognitive architecture (Sweller, van Merriënboer, & Paas, 2019). A major premise of the theory is that working memory load from cognitive processes is decreased when domain specific schemas are activated from long term memory. Comprehension, schema construction, schema automation, and problem solving in working memory often create high cognitive load. Hence, schemas transported from long term memory into working memory support learning and transfer of learning (Ginns & Leppin, 2019). One of the key developments from CLT has been the Four-Component Instructional Design (4C/ID) Model generated from evolutionary theorizing (Geary, 2008; Ginns & Leppink, 2019). Since its creation, the 4C/ID Model has been successfully applied to instruction that requires the learning of complex tasks. Van Merriënboer, Kester, and Paas (2006) defined a complex task as having many different solutions, real world connections, requiring time to learn, and as creating a high cognitive load. Based on this definition, the instruction and learning of mathematics is a complex task. For example, different solutions are algebraic, analytic, numeric, and graphic. Relative to real world connections, mathematics is one of the domains in the broader science, technology, engineering, mathematics (STEM) field and is regarded as the language of the sciences. Regarding taking time to learn and creating a high load on learner's cognitive systems, mathematics teachers deal with the tension between covering all the required standards and taking the time to teach for understanding. Teachers face challenging decisions about instructional approaches, materials, productive struggle, and the amount of classroom time spent on various standards. Better models for instruction that support transfer of learning could help teachers improve instructional decision making. Although the 4C/ID Model has been used in secondary mathematics education (Sarfo, & Elen, 2007; Wade, 2011), it has never been confirmed as

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

a mathematical instructional theory. The purpose of this research report is to present an empirical confirmation of the 4C/ID Model, using data from the Factors Influencing College Success in Mathematics (FICSMath) project from Harvard University.

## FICSMath Project

The Factors Influencing College Success in Mathematics (FICSMath) Project remains the largest and most recent national study of the secondary-tertiary transition in mathematics. Towards the beginning of the 2009 fall semester, college freshmen in single variable calculus courses across the United States (US) responded to questions on the FICSMath survey regarding educational experiences in their last high school mathematics course. Professors secured students' completed surveys until the end of the semester and recorded final grades for each student on their respective survey before returning them to Harvard University. A total of 10,492 surveys were collected, and from this sample 5,985 students had taken either precalculus (n=2,326), or any level of high school calculus (n=3,659) as their most recent high school mathematics course. The 4C/ID Model appears appropriate to use as a theoretical lens through which to view secondary preparation for college calculus because the components of the model explicitly consider instruction to support transfer of learning (van Merriënboer, Kester, Paas, 2006).

## The 4C/ID Model

Van Merriënboer and other cognitive load theorists developed the 4C/ID Model in the early 1990s under the premise that instruction for complex tasks should be combined with methods that have been shown to enhance transfer of learning (Van Merriënboer, Kirschner, Kester, 2003;

Van Merriënboer, Clark, & de Croock, 2002). Transfer is required when prior learning must be recalled to support the learning of new tasks. Vertical transfer is required, for example, to transfer knowledge from the high school mathematics to college calculus. The model was not designed specifically for mathematics instruction, but generally for learning environments where complex problems are the basis of instruction and transfer of learning is the goal.

The 4C/ID Model employs human cognitive architecture from cognitive load theory (Sweller, 2008). The assumptions are that working memory is limited in space and duration while there appears to be no limit of either in long-term memory. The three sources of working memory load are assumed to be: (a) extraneous cognitive load coming from how the material is presented during instruction; (b) intrinsic cognitive load coming from element interactivity, or the interaction of the interconnected parts of the content; and (c) germane cognitive load, which sends and hooks new processed and encoded information into long term memory to be connected with existing schemas. Once information has been processed and connected within the learners' schemas in long term memory, it can then be brought back into working memory as a chunk of knowledge to help process more new content. Integration of new content into schemas makes learning more efficient as it lowers the demands on working memory and supports the learning of complex tasks.

### Model Components

The 4C/ID Model incorporates four components: Learning Task, Support, Procedure, and Part-Task Components. These come from theorizing how to instruct a complex task to enable working and long-term memory to develop, retain, and recall comprehension, schema construction, schema automation, and problem solving. Figure 1 shows how Van Merriënboer, Kester, and Paas (2006) theorized the model. Each of these components needs attention during precalculus and calculus instruction. The neglect of any one of them could prohibit learning and/or transfer of learning. As such, the components are discussed specifically regarding the instruction of mathematics during the secondary-tertiary transition.

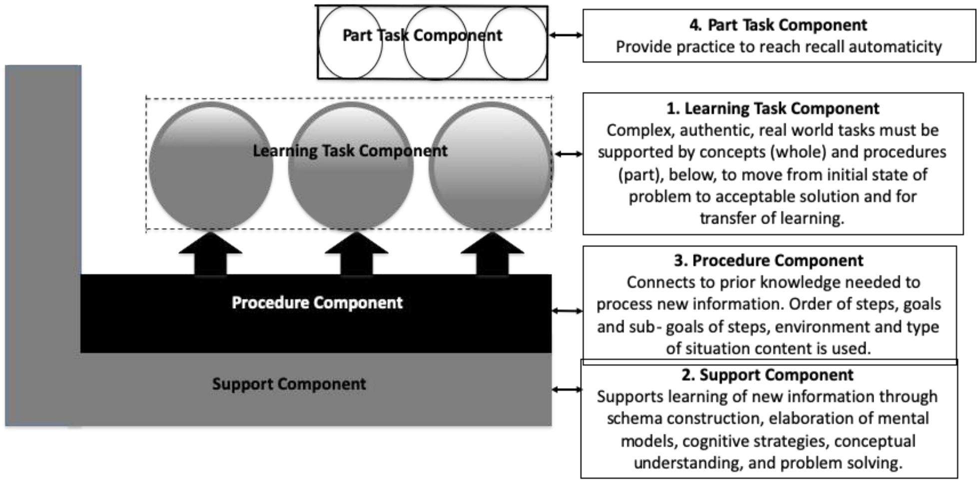


**Figure 1: The theorized general 4C/ID Model (modified from van Merriënboer, Clark, De Crook, [2002])**

1. The Learning Task Component is modeled to engage learners in meaningful problem-solving tasks. Working with real world problems, often integrated into mathematics to motivate learning (Beswick, 2010), requires mental processes to move from the initial state of the problem to an acceptable solution (van Merriënboer et al., 2003). Engagement in higher-level tasks during mathematics instruction increases students' engagement with mathematical ideas (Boaler and Staples, 2008). Such tasks include high element interactivity, which occurs because of the interacting parts of the mathematics that must be addressed during problem solving. Element interactivity is inherent in secondary preparation for college calculus because of the many interacting mathematical concepts involved in precalculus and calculus problem solving.
2. The Support Component undergirds the Learning Task Component and includes conceptual understanding, reasoning of new information, problem solving, and cognitive assessment (van Merriënboer, Clark, de Croock, 2002).
3. The Procedure Component integrates examples, hooks to previous learning or schemas from long-term memory, which supports the processing of complex ideas. These are important instructional practices in mathematics (Wade, Sonnert, Sadler, & Hazari, 2017; Wade, Cimbricz, Sonnert, Gruver, & Sadler, 2019). This is, first, because mathematics is abstract, and reasoning is required to understand abstract information (Russell, 1999). Another reason is that, when strategies are recalled from long term memory, it is common for mistakes in the problem-solving process to occur. Yet, with guidance, students can learn from their mistakes. This process is referred to as flawed reasoning and is believed to be an important part of learning mathematics (Russell, 1999).
4. The Part Task Component models instruction working towards students developing automaticity. This means that specific tasks from previous learning can occur with little effort, requiring little conscious monitoring and few cognitive resources (Feldon, 2007). The part-task component is included in the 4C/ID Model because there are times that instruction allows repeated practice of information to the point of automaticity. Depending on where the learner is in understanding whole concepts first, this can both benefit or hinder the meaningful learning of mathematics. For example, if a student has not learned the concepts but practices procedures, the result is often what Skemp (2015) refers to as instrumental understanding or rules without reason. The goal is relational understanding, which Skemp

(2015) defined as knowing what to do and why, which requires understanding concepts as well as procedures.

## Research Question and Method

Can the 4C/ID Model be empirically validated for mathematics instruction for the secondary-tertiary level using data from the FICSMath Project and confirmatory factor analysis?

### Confirmatory Factor Analysis Model

Freshmen respondents in single variable college calculus courses from large, medium, and small 2- and 4-year institutions from across the nation reported instructional experiences from their senior-level high school precalculus or calculus courses (n=5,985). The percent of missing value cases were small (between 1.3% and 4.9%), yet multiple imputation was computed to create a small number of copies of the dataset, with each having missing values suitably imputed. Each complete dataset was analyzed independently and estimates of parameters of interest were averaged across the copies to provide a single estimate (Royston, 2004). In the end, the model reported 6,146 cases, a 2.6% increase from the 5985 respondents included in the model. Then confirmatory factor analysis (CFA) was used to test the extent to which these variables related to the underlying constructs of the 4C/ID Model.

CFA is theory driven, so we began by analyzing the theoretical relationships among the observed and unobserved, or latent, variables (Schreiber, Nora, Stage, Barlow, & King, 2006). The observed variables (Figure 2 rectangles) are intercorrelated secondary instructional experiences reported by single variable college calculus students who completed the FICSMath survey. The unobserved variables (Figure 2 large ovals) are factors that account for correlations among the observed variables (Brown & Moore, 2012) that theoretically aligned to the 4C/ID constructs. We identified instructional experiences that provided: (a) complex mathematical tasks (Learning Task Component, n=4); (b) an overview of whole task mathematical concepts (Support Component, n=17); (c) support for the processing of mathematics, the use of algorithms, and graphing (Procedural Component, n=16); and (d) opportunities for practice (Part-Task Component, n=13). Figure 2 shows the number of observed variables that converged and survived CFA. The loadings to the right of the large ovals show the correlations between the components while loadings to the right of the rectangles show the correlations between the observed variables to each component. The small ovals connected to each rectangle on the left show the errors associated with the observed variables in the model. The loadings to the left of the errors show their correlations while one shows the correlation between the error for emphasis on vocabulary and the support component (discussed later).

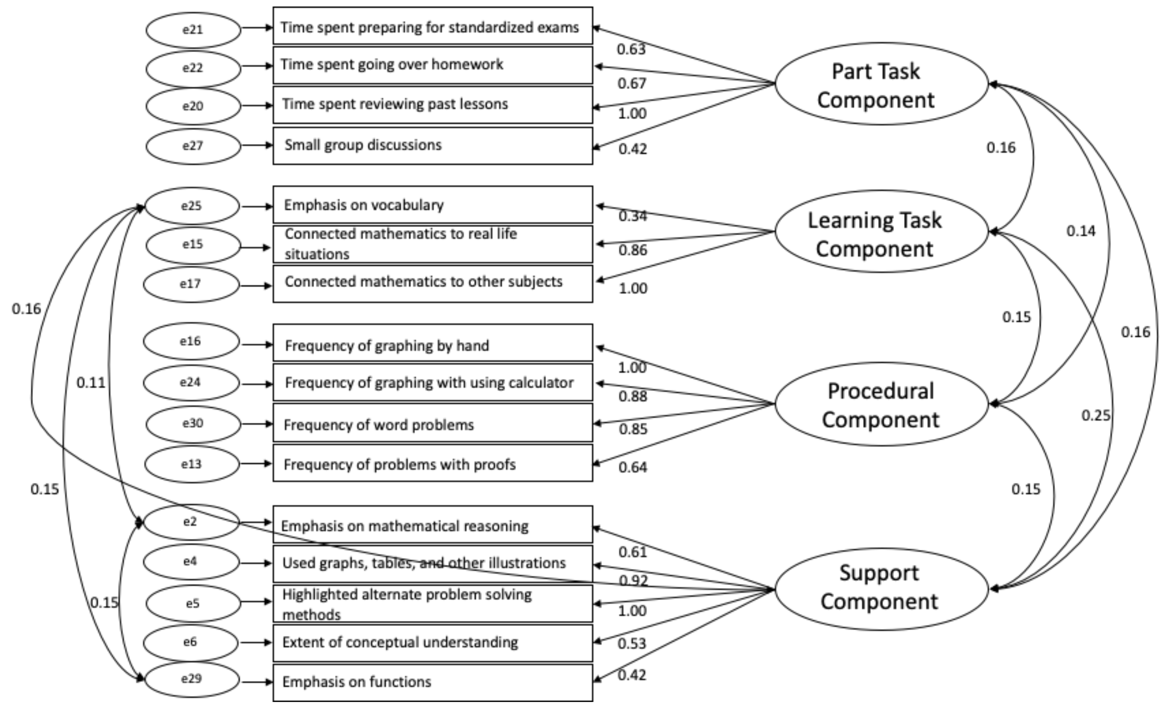


**Figure 2: The 4C/ID *Math* Model Confirmed using Confirmatory Factor Analysis.**

The large FICSMath sample size (n=5,985) allows assumptions of normality of data and increases the power of a hypothesis test. This large sample size, however, did limit some of the CFA measures that can be reported. For example, the chi square test, Normed Fit Index (NFI), and Tucker Lewis Index (TLI) are typically reported in CFA models, but these are preferable measures for smaller data sets. As shown in Table 1, the Comparative Fit Index (CFI) and the Root Mean Square Error Approximation (REMSA) confirm the components in the 4C/ID Model have meaningful relationships with the observed variables in the FICSMath dataset.

**Table 1: Measures of CFA Reported, Accepted Cut-off Scores for Significance, Results of the 4C/ID Math CFA Model with Notes for Clarity.**

| CFA Measure | Cutoff for Significance | Model Value | Notes |
|---|---|---|---|
| Comparative Fit Index (CFI) | CFI > 0.90 | 0.907* | Compares the fit of a target model to the fit of a null hypothesis model. |
| Root Mean Square Error Approximation (RMSEA) | RMSEA < 0.08 | 0.050* | A parsimony-adjusted index. Values closer to 0 represent a good fit. |

*Significant finding. (See Parry (no date); Brown & Moore, 2012).

Figure 3 shows a representation of the confirmed 4C/ID Model, now referred to as the 4C/ID*Math* Model. The constructs have been renamed to align closer with the field of mathematics education. These components are now:

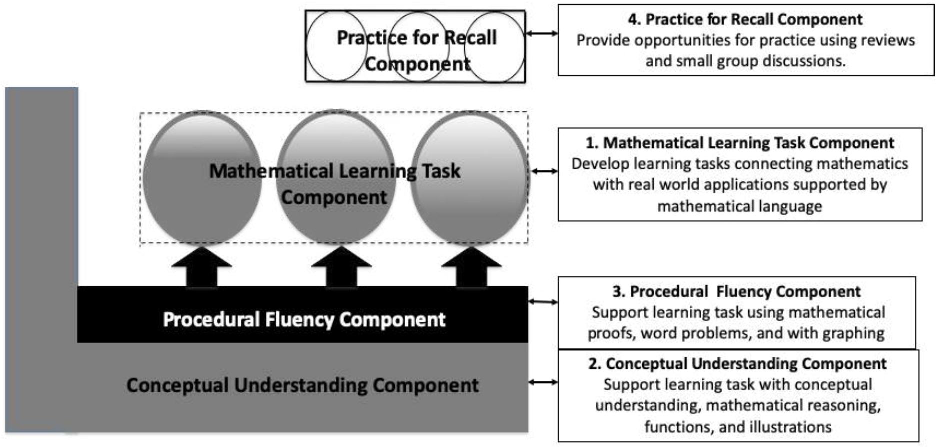


**Figure 3: The 4C/ID*Math* Model for Secondary-Tertiary Mathematics (modified from van Merriënboer, Clark, De Crook, [2002])**

1. The Mathematical Learning Task Component is the new name for the Learning Task Component. This is where whole tasks should be presented to avoid the transfer paradox. The transfer paradox is described as occurring when instruction breaks apart concepts to minimize the necessary time-on-task. This type of instruction has been shown to have a positive effect on short term retention for performance on tests, but not on transfer of learning (van Merriënboer et al., 2006).
2. The Conceptual Understanding Component is the new name for the Support Component. This name change aligns with what teachers who were identified as teaching for high conceptual understanding on the FICSMath survey concretely did to teach for conceptual understanding (Wade, Sonnert, Sadler, Hazari (2017). This study showed that teaching functions and mathematical reasoning was highly correlated with conceptual understanding.
3. The Procedural Fluency Component is the new name for the Procedure Component. Star (2005) presented thinking flexibly with mathematics as an indicator of deep procedural knowledge. To generate graphs, students must be able to think flexibly across the connections between equations and algorithms to points on various graphing planes. Mathematical proofs require meaningful connections across relevant mathematical relationships, which requires thinking flexibly with those relationships (Williams-Pierce et al., 2017).
4. Practice for Recall Component is the new name for the Part-Task Component. Van Merriënboer, Kester, and Paas (2006) stated that part-task practice may provide additional practice needed to develop knowledge elements that allow the learner to perform routine aspects at a high level of automaticity. In mathematics education, this is better understood simply as practice for recall.

## Limitations and Future Work

One weakness of the study may be that the FICSMath Project is from 2009, yet this project remains the most recent national study on secondary preparation for college calculus success. Until the FICSMath project can be replaced by another large-scale national study, the national representation and sample size strength of the project warrants its continued use. Additionally, the 4C/ID*Math* Model is confirmed for students in the secondary-tertiary transition who took either precalculus or calculus as their last mathematics course before entering into single variable college calculus. More research is needed to confirm the model at different levels of mathematical instruction, such as for algebra or geometry. Lastly, how the 4C/ID*Math* Model actually predicts performance in single variable college calculus needs to be investigated. The focus of this paper was to confirm the 4C/ID Model and then modify it to be more user friendly for mathematics teachers.

## Discussion

The theoretical perspective of the 4C/ID Model is that instruction of complex tasks should be guided by principles that reinforce learning and transfer of learning. The 4C/ID Model theorizes the Support Component as concepts that structure the learning of complex tasks and the Procedure Component as connecting prior learning, the order of steps and context for use. Both of these components undergird instruction of a complex task, which is represented as the Learning Task Component. The Part Task Component symbolizes the use of automatized information that requires little to no cognitive load in working memory. Van Merriënboer et al. (2006) state the 4C/ID Model was designed to focus instruction on whole tasks and claims breaking apart concepts to minimize time-on-task has a positive effect on short term retention for performance on tests, but not on transfer of learning. This was theorized as the transfer paradox. Skemp (2006) presented similar ideas in mathematics education through relational and instrumental understanding. Relational understanding comes from instruction that focuses on knowing what to do and why while instrumental understanding was conceived as instruction that focused on rules without reason. It was claimed that high school teachers often adopt a two-track strategy of instruction where they spend some time on drill and practice, providing for skills and facts, and some time on developing and integrating understandings (Skemp, 2006). Based on the 4C/ID Model, drill and practice can develop automaticity but does not reinforce learning for transfer. These similarities indicate the 4C/ID Model to be a good fit with mathematics education. The empirical confirmation of the 4C/ID Model using the FICSMath Project resulted in the 4C/ID*Math* Model for secondary-tertiary mathematics instruction. The 4C/ID*Math* Model confirms the importance of generating the learning task first then considering the concepts and procedures needed for learning and transfer of learning. Each of the components for the 4C/ID*Math* Model is discussed below relative to how this model can be used in precalculus and calculus secondary-tertiary mathematics instruction.

1. The Mathematical Learning Task Component represents complex tasks that must be considered as a whole to support transfer of learning. Instruction that breaks apart concepts to minimize time for learning has been shown to have a positive effect on short term retention but not on transfer of learning (van Merriënboer et al., 2006). Considering complex tasks and how to present the many interacting elements as a whole concept first is important, especially in mathematics where transfer of learning is critical. As seen in Figure 2, this component includes the emphasis on (mathematical) vocabulary item. The vocabulary item was originally mapped to the Support Component, but the CFA model was not valid. When the vocabulary item was moved to the Learning Task Component, the error associated with the item was too high. After correlating the vocabulary item error term with the Support Component, the 4C/ID Model converged. This correlation indicates the vocabulary term is essential to both, the Mathematical Learning Task and the Support Component. Tall (2004) stated real world representations require the sophistication of language to support abstract concepts in formal mathematics. After determining what standards and elements are to be instructed, focus should then be placed on the language required to present the content and how to connect the mathematics to real world applications and other subjects.
2. The Conceptual Understanding Component emphasizes conceptual understanding, mathematical reasoning, functions, illustrations, and alternate problem-solving methods necessary to support learning mathematical content. Wade, Sonnert, Sadler, and Hazari (2017) showed mathematical reasoning and emphasis on functions to be part of the construct

that described what teachers did to teach for conceptual understanding. This component aligns well with the field of mathematics education.

3. The Procedural Fluency Component demonstrates hooking previous learning, or schemas, from long-term memory to concepts presented in the Conceptual Understanding Component. Graphing functions and equations require modeling mathematics both by hand and, in the secondary mathematics classroom, the graphing calculator. Most secondary mathematics standardized exams, including AP exams, require the use of a graphing calculator but most single variable college calculus courses do not allow their use in class or on exams. This implies the importance of students understanding the mathematical procedures even if they have a graphing calculator available. Mathematical proofs, independent of the format, require justification from prior learning and are an important part of the secondary-tertiary transition. At the tertiary level, proofs tend to be longer, more complex, and require more mathematical insight than at the secondary level (Selden, 2011). Many students are not well prepared for the types of proofs they will be exposed to in college calculus (Bressoud, 2009). It could be that incorporating more proofs into secondary precalculus and calculus courses may reduce some of the transition struggles for students in college calculus.
4. The Practice for Recall Component illustrates that opportunities for practice using reviews and small group discussions are beneficial for developing automatic recall.

It is our hope that the confirmation of the 4C/ID Model, leading to the 4C/ID*Math* Model, brings this instructional framework into the purview of secondary mathematics teachers and mathematics professors who teach students in the secondary-tertiary transition. Better preparation for single variable college calculus is important because this is the first mathematics course that is commonly required in all STEM majors.

## Acknowledgment

This research was supported by Grant No. 0813702 from the National Science Foundation. Any opinions, findings, and conclusions in this article are the authors' and do not necessarily reflect the views of the National Science Foundation. Without the excellent contributions of many people, the FICSMath project would not have been possible. We thank the members of the FICSMath team, the mathematics educators who provided advice or counsel on this project, and the many college students and professors who participated in the survey.

## References

Beswick, K. (2011). Putting context in context: An examination of the evidence for the benefits of 'contextualized' tasks. *International journal of science and mathematics education*, *9*(2), 367-390.

Boaler, Jo, and Megan Staples. "Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School." *Teachers College Record* 110, no. 3 (2008): 608–45.

Bressoud, D. M. (2009). *AP calculus: What we know* MAA, June 2009. Brown, C., & Smith, M. (1997). Supporting the development of mathematics pedagogy. *Mathematics Teacher*, 90(2), 138–143.

Brown, T. A., & Moore, M. T. (2012). Confirmatory factor analysis. *Handbook of structural equation modeling*, 361-379.

Geary, D. (2008). An evolutionarily informed education science. *Educational Psychologist, 43*(4), 179–195.

Kline, R. B. (2015). Principles and practice of structural equation modeling. Guilford publications.

Feldon, D. (2007). Cognitive load and classroom teaching: the double-edged sword of automaticity. Educational psychologist, 42(3), 123-137.

Ginns, P., & Leppink, J. (2019). *Special Issue on Cognitive Load Theory. Educational Psychology Review, 1-5.*

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.

Parry, S. Fit statistics commonly reported for CFA and SEM. Retrieved from https://www.hrstud.unizg.hr/_download/repository/SEM_fit.pdf

Royston, P. (2004). Multiple imputation of missing values. *The Stata Journal*, *4*(3), 227-241.
Russell, S. J. (1999). Mathematical reasoning in the elementary grades. *Developing mathematical reasoning in grades K-12. NCTM 1999 Yearbook*. (pp. 1-12).
Sarfo, F. K., & Elen, J. (2007). Developing technical expertise in secondary technical schools: The effect of 4C/ID learning environments. *Learning Environments Research*, *10*(3), 207-221.
Schreiber, J. B., Nora, A., Stage, F. K., Barlow, E. A., & King, J. (2006). Reporting structural equation modeling and confirmatory factor analysis results: A review. *The Journal of educational research*, *99*(6), 323-338.
Selden, A. (2011). Transitions and proof and proving at tertiary level. In *Proof and proving in mathematics education* (pp. 391-420). Springer, Dordrecht.
Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for research in mathematics education*, 404-411.
Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics teaching in the middle school*, *12*(2), 88-95.
Sweller, J., van Merriënboer, J. J., & Pass, F. (2019). Cognitive architecture and instructional design: 20 years later. *Educational Psychology Review*, 1-32.
Sweller, J. (2008). Human cognitive architecture. Handbook of research on educational communications and technology, 369-381.
Van Merriënboer, J.G., Kester, L., & Paas, F. (2006). Teaching complex rather than simple tasks: Balancing intrinsic and germane load to enhance transfer of learning. *Applied Cognitive Psychology, 20*(3), 343-352.
Van Merrienboer, J.G., Kirschner, P., Kester, L. (2003). Taking the load off a learner's mind: Instructional design for complex learning. Educational psychologist, 38(1), 5-13.
Van Merrienboer, J.G., Clark, R.E., & de Croock, M.B. (2002). Blue-prints for complex learning: The 4C/ID Model. *Educational technology, research and development*, 50(2), 39-64.
Vandewaetere, M., Manhaeve, D., Aertgeerts, B., Clarebout, G., Van Merriënboer, J. J., & Roex, (2015). 4C/ID in medical education: How to design an educational program based on whole-task learning: AMEE Guide No. 93. *Medical teacher*, *37*(1), 4-20.
Wade, C. H., Cimbricz, S. K., Sonnert, G., Gruver, M., & Sadler, P. M. (2018). The Secondary-Tertiary Transition in Mathematics: What High School Teachers Do to Prepare Students for Future Success in College-Level Calculus. *Journal of Mathematics Education at Teachers College*, *9*(2).
Wade, C. H., Sonnert, G., Sadler, P. M., & Hazari, Z. (2017). Instructional experiences that align with conceptual understanding in the transition from high school mathematics to college calculus. *American Secondary Education*.
Wade, C. (2011). Secondary preparation for single variable college calculus: Significant pedagogies used to revise the four component instructional design model.
Williams-Pierce, C., Pier, E. L., Walkington, C., Boncoddo, R., Clinton, V., Alibali, M. W., & Nathan, M. J. (2017). What we say and how we do: action, gesture, and language in proving. *Journal for Research in Mathematics Education*, *48*(3), 248-260.
Wolf, E. J., Harrington, K. M., Clark, S. L., & Miller, M. W. (2013). Sample size requirements for structural equation models: An evaluation of power, bias, and solution propriety. *Educational and psychological measurement*, *73*(6), 913-934.