SUPPORTING ENGLISH LEARNER STUDENTS' DEVELOPMENT OF THE MATHEMATICS REGISTER THROUGH AN INSTRUCTIONAL INTERVENTION

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We describe an instructional intervention designed to help teachers engage English learner (EL) students in mathematical problem solving and learn the mathematics register. The "Discursive Assessment Protocol" (DAP) integrates Pólya's classic problem solving framework with research-based instructional strategies that benefit EL students. The research-based instructional strategies are grounded in theories of academic language development. A sample problem-solving episode is provided that demonstrates how an EL student wrote a "multiplication story" involving fractions and what we learned from using the DAP to support him and other EL students develop the mathematics register in English. Among the implications of this study is the value of selecting tasks that are not only worthwhile mathematically, but worthwhile in that they have potential to develop students' mathematics register.

Keywords: Equity and Diversity, Marginalized communities, Problem Solving.

Schools are struggling to meet the needs of English Learners (ELs) in the United States (Borjian, 2008; Valenzuela, 2005). ELs largely enter U.S. schools performing below English Proficient (EP) students in core academic subjects (Abedi & Gándara, 2006) and dropout rates for ELs are considerably higher than EP students (Borjian, 2008; Kanno & Cromley, 2013). Schools experiencing an influx of EL students must adjust to meet these students' educational needs (Barrio, 2017; Irizarry, 2011). In this paper, we describe an instructional intervention designed to help teachers support the mathematical learning of their EL students and how it informed instruction during a problem-solving episode. The "Discursive Assessment Protocol" (DAP) integrates Pólya's (1945/1986) classic problem solving framework with research-based instructional strategies that benefit ELs. The research-based instructional strategies are grounded in theories of academic language development that afford EL students repeated and consistent opportunities to express their mathematical ideas and negotiate meaning with others (Moschkovich, 2013; 2015). The integrated design of the DAP is intended to guide teachers to provide their students with needed supports to learn and use the mathematics register during problem solving episodes.

Since the fall of 2019, we have been working with "Ms. Ware," a 5th grade teacher in an urban school district. A goal of this work has been to examine the DAP as an instructional intervention in Ms. Ware's mathematics classes to understand how effectively it guides her to elicit students' mathematical reasoning and develop their use of the mathematics register in English during problem-solving episodes. In all of her classes, Ms. Ware teaches a high percentage of ELs. She is fluent in Spanish and is devoted to providing her EL students with a high quality education in mathematics. The research question we explore here is: How does the DAP used during problem-solving episodes inform how to support EL students to develop the mathematics register in English?

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Background on the Discursive Assessment Protocol

Early iterations of the DAP built upon and were an extension of a clinical interview protocol (see Kitchen & Wilson, 2004; Kulm, Wilson, & Kitchen, 2005). The DAP was designed for use with individual ELs or groups of EL students during mathematics problem-solving episodes, but can also be used with the general student population. During piloting of the DAP with middle school students over the course of two years (2007-09), the DAP guided teachers to provide students with opportunities to ask questions, to be creative, to test and revise their solution strategies, and to explore mathematical ideas deeply (see Kitchen, Burr, & Castellón, 2010; Castellón, Burr, & Kitchen, 2011). Such instruction is a clear departure from instruction historically found in schools that serve high percentages of low-income EL students in which the memorization of math facts, algorithms, vocabulary, and procedures are the focal point of instruction (Kitchen, DePree, Celedón-Pattichis, & Brinkerhoff, 2007; Moschkovich, 2013). Moreover, the DAP is intended to help teachers provide students with opportunities to make sense of the language demands of mathematical problems as well as to provide scaffolded supports for EL students to engage in mathematical discourse to explain their ideas and to listen to and make sense of the ideas of others. As students engage in mathematical discourse, they build on their prior experiences and knowledge to achieve more advanced understandings of mathematical concepts (Ryve, 2011).

Incorporating Pólya's (1945/1986) four-stage problem solving framework, the DAP is designed to be administered during problem-solving episodes involving worthwhile mathematics tasks, ensuring that students have something to talk about (Silver & Smith, 1996). In the example that we provide here, we used a performance assessment task (referred to simply as "task" throughout) that is publicly accessible for free through the Illustrative Mathematics (IM) project. Rich tasks such as performance assessment tasks "engage students in thinking and reasoning about important mathematical ideas" (Franke, Kazemi, & Battey, 2007, p. 234). Though the use of such tasks does not guarantee high-level student responses, cognitively demanding tasks provide the means for teachers to engage students in mathematical discourse in which students are actively sharing their thinking, comparing their solution strategies, making conjectures, and generalizing (Silver & Smith, 1996).

An important goal of instruction for EL students should be amplifying rather than complexifying English language speech or text (Zwiers et al., 2017). This entails providing students with multiple opportunities to understand mathematical ideas and terms by providing support for learning with concrete materials such as manipulatives and mathematical models, engaging students in think-alouds, and using culturally relevant and authentic contexts. ELs need repeated opportunities to understand the problem at hand, not only because English is a second language, but because the learning of mathematics is embedded within the linguistic patterns of academic language development. Academic language has been defined as the linguistic expectations of students to learn, speak, read and write about academic subjects such as mathematics (Schleppegrell, 2004). Discipline-specific registers can further refine academic language. Described as words, expressions, and meanings specific to mathematics (Secada, 1992), the mathematics register is the disciplinary-specific reading, writing, listening and speaking norms of content teaching and learning that is more complex than everyday English. It is helpful to think of the academic register as a series of resources that promote meaning making, or a set of linguistic features, such as words, symbols, and forms (Schleppegrell, 2004). A unique feature of the DAP is that instructional strategies designed to support the development of EL students' mathematics register, also referred to as English as a Second Language (ESL) instructional strategies, are incorporated throughout its four stages, such as acknowledging and using gestures, integrating cognates, revoicing, and incorporating graphic organizers (see Figure 1 below).

1. Understand the problem	
Check for understanding	- Students underline important information in problem - Teachers ask: "What is the problem asking you to do? What do you know that can help you figure this out?" - Students define new words and begin using them in sentences.
Teacher deliberately incorporates ESL strategies	- Students use a picture, diagram, or some type of mathematical representation to concretely model problem.
Teacher maintains high expectations and recognizes students' intellectual assets	- Teachers look for opportunities to highlight students' mathematical ideas with other students.
2. Create a plan to solve the problem	
Students create plan to solve problem	- Teachers ask: "What strategy, representation or tool will work best to solve the problem?" - Teachers assess student understanding of their plan.
Teacher deliberately incorporates ESL strategies	- Teachers integrate graphic organizers and mathematical models during small group instruction and discourse.
3. Carry out the plan to solve the problem	
Teacher engages students in mathematical discourse and meaning making	- Teachers engage whole class in mathematical discourse and asks questions while highlighting student work. - Teachers integrate the mathematics register in discourse and instruction.
Teacher continues to use deliberate ESL strategies	- Teachers use gestures, cognates, revoicing, graphic organizers and mathematical models.
Students refine and revise their solutions	- Teachers do not need to be overly concerned in this stage about students' production of "correct" English.
4. Looking back	
Students reflect on their solutions	- Teachers ask: "Does your solution make sense? How do you know? What questions do you still have at this point?"
Teacher works to help students use the formalized mathematics register	- Students write up their final solution to the problem using the mathematics register.

Figure 1: The Discursive Assessment Protocol (DAP)

The first stage of the DAP involves understanding the task/problem (Pólya, 1945/1986). In this stage, Pólya advocates that students consider a picture, diagram, or some type of mathematical model that could be helpful for solving the problem. Modeling helps EL students learn the mathematics register by touching the objects that represent mathematical ideas and repeatedly hearing and then repeating the words represented by these objects (Garrison & Mora, 2005). After developing a mathematical model to make sense of the problem, students share their ideas with peers to solicit feedback and modify their models. To ensure ELs have repeated opportunities to understand the problem at hand, teachers ask questions during this stage such as "What is the problem asking you to do?" and "How are you going to figure this out?" In this stage, teachers need to define words used in the problem under consideration. In the second stage, students devise a plan to solve the problem (Pólya, 1945/1986). Working in small groups, students share their ideas with peers and their teacher to get feedback on their solution strategies. This process ensures that students have opportunities to reflect upon their problem-solving strategy to determine whether the strategy makes sense. Students need support in this stage to develop self-regulation strategies such as devoting significant time to analyze and plan how to attack the problem similar to accomplished problem solvers (Schoenfeld, 1985).

In the third stage, students carry out their plan to solve the problem (Pólya, 1945/1986). Students have opportunities to share their mathematical thinking with peers and their teacher through mathematical discourse. When possible, the teacher seeks to integrate the mathematics register in discourse and instruction, though does not need to be overly concerned with students using "correct" English. A key in this stage is that the teacher asks meaningful questions and actively works to highlight and build on students' ideas to support students reflecting on their mathematical thinking and errors (Schoenfeld 1985). Instruction should leverage ELs' knowledge in their first language as a means to help them comprehend a second language (Cummins, 2000). To support ELs in particular, the teacher "re-voices" students' explanations, references students' mathematical ideas, and asks clarifying questions. In this manner, the DAP functions as a formative assessment tool, supporting teachers to examine, understand, and leverage students' mathematical ideas and thinking as a means to inform their instruction (Kitchen, 2014). In the fourth stage, students look back at their solutions and check their results (Pólya, 1945/1986). In this stage, the teacher asks: "Does your solution make sense? How do you know? What questions do you still have at this point?" In addition to reviewing and checking their answers, ELs need opportunities to explain their ideas using the mathematics register. In this stage, students explain their problem solutions in writing with the expectation that they will include the mathematics register in their write ups. Having had time to think about, solve and revise their solutions also means students' anxiety level, the affective filter (Krashen, 2009), has been lowered and ELs may have more confidence explaining their ideas in writing.

Research Methodology

Starting in the fall of 2019, our research team (Richard, Libni and Karla) has been collaborating with Ms. Ware to implement the DAP during problem-solving episodes with her two 5th grade mathematics classes. Both of these classes have a high percentage of ELs (20% or more). To date, we have co-taught with Ms. Ware during problem-solving episodes on four occasions. We employed a team teaching approach to co-teach in which instruction was divided up among the four of us (Cook & Friend, 1995; Sileo & van Garderen, 2010). Each problem-solving episode typically lasted between 40 and 60 minutes and involved students solving a performance assessment task. During each episode, we worked to follow the four stages of the DAP as students solved a given task. Primarily in the second and third stages of DAP implementation, all three members of the research team circulated throughout the classroom with Ms. Ware, asking individual and groups of students questions engaging in discourse. Prior to each problem-solving episode, we planned how we hoped to co-teach during the episode, identified questions to ask, and discussed English words and phrases that we hoped to develop during instruction to support EL students' emerging mathematics register. During the problem-solving episodes, we videotaped Ms. Ware and students who had provided consent. At the conclusion of these episodes, we collected all the work students had created.

To illuminate how the DAP informs instruction, a sample student solution to a performance assessment task is provided. Specifically, we highlight how one EL student, "Fernando," solved a given task and how his solution informed us vis-à-vis how to support Fernando and other EL students to more fluently construct English sentences in the "multiplication stories" that they devised. The data used in the example provided came from copies of collected student work samples and from videotapes made during the problem-solving episode. Student work samples and videotapes were interpreted using interpretative methods (Creswell, 2014). The student work samples were initially read or viewed as a whole, followed by a period of open coding to reflect upon and clarify how students were solving a given task and how they used the mathematics register to express their solutions. An iterative process of coding, reflecting upon, and then clarifying what we learned from reviewing student work samples then took place (Miles, Huberman & Saldana, 2013). This process went through multiple revisions as the data were repeatedly read and reviewed to check the consistency of findings. This process continued until either no new categories were developed or consistency was achieved. After we established how to characterize students' solutions to tasks posed, we searched for commonalities and differences across these solutions to further examine how the DAP could be used to inform instruction intended to support the development of EL students' mathematics register. We went through a similar process when reviewing videotapes of problem-solving episodes.

Research Findings

We now offer an example of how through the use of the DAP, we gained insight into how to support the development of an EL student's mathematics register in English during a problem-solving episode. Immediately prior to the problem-solving episode, Karla and Libni led a brief lesson on fractions to Ms. Ware's students. They had students identify unit fractions in both columns and rows of a rectangular whole similar to the rectangular whole displayed in Figure 2. In several exercises, students identified equivalent fractions in diagrams given to them such as 1/5 and 2/10. Karla and Libni also had students identify the fraction created when two of these fractions overlapped, something Ms. Ware had been doing with her students for at least a week prior to this lesson. In addition, students began identifying an equation that could be derived through fraction multiplication. The purpose was to emphasize the meaning of fractions as operator (e.g., 1/4 of 1/3). Following this brief lesson, the four stages of the DAP were administered during implementation of the following IM task shown in Figure 2.

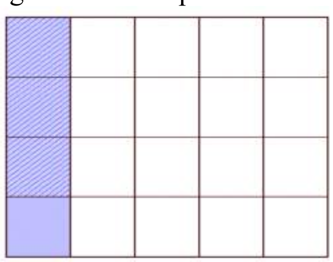
The diagram below represents one whole. 	Problem: Write a multiplication story that could be solved using this diagram with its two types of shading. Explain how your story context relates to the diagram provided (http://tasks.illustrativemathematics.org/content-standards/5/NF/B/4/tasks/2075).

Figure 2: Task Implemented with the DAP

Ms. Ware introduced the task by asking three different students to read the problem out loud to the whole class and give brief explanations about what the problem was asking as a means to check for understanding (first stage of the DAP: Understand the problem). In addition, as planned prior to the lesson, Ms. Ware began asking questions we had collaboratively identified such as *What is the problem asking you to do? What is the whole? What is a multiplication story?* She also checked for understanding of the terms "two types of shading" and "relates." Once Ms. Ware was satisfied that her students, for the most part, understood the task because a number of them could express what the task was asking of them, she moved on to the second stage of the DAP.

Initially working alone, students started devising a plan to solve the task (second stage of the DAP: Create a plan to solve the problem). It was in this stage that Fernando devised a multiplication story involving videogames. In his written solution, he started by making sense of the diagram given in Figure 2; he identified the whole, circling the entire diagram and writing "The Whole." He also identified both of the fractions represented in the diagram (3/4 and 1/5). Lastly, Fernando wrote the following expression that he believed was represented in the diagram: 3/4 x 1/5.

In his write-up (stage 3 of DAP, Carry out plan), Fernando created a story that was mathematically sophisticated involving videogames:

> "Sam got to play videogames 1/5 of an hour. After he did his homework he got to play 3/4 of the 1/5 that he played. How much does he play after homework?"

During the third stage of the DAP, teachers engage the whole class in mathematical discourse and ask questions while highlighting student work. Fernando shared his story in his small group. At one point, he also responded to another student's story during a whole class discussion that took place. The language that he used in the context of his story is specific to the operator concept of fraction (e.g., *"he got to play 3/4 of the 1/5")*. The operator subconstruct has two different interpretations, as stretcher/shrinker and as a duplicator/partition-reducer. The difference between the two is that with stretcher/shrinker, the transformation of the fraction results in the same number of units of different size (e.g. 3/4 should be interpreted as 3 x [1/4 of a unit]), and with the duplicator/partition-reducer the fraction result elicits a different number of units of the same size (e.g. 1/4 x [3 units]) (Charalambous & PittaPantazi, 2007). The operator subconstruct can also be considered a function, a set of operations that need to be done to get a result (Lamon, 2007). In this case, Fernando used the duplicator/partition-reducer interpretation.

While Fernando's story is mathematically sophisticated, the clarity of the story could be improved in at least two ways. First, he could modify the second sentence to read, "*After he did his homework, he got to play 3/4 of the 1/5* of an hour *that he had already played*." The inclusion of the phrase "of an hour" in Fernando's sentence clarifies the amount of time that he originally played videogames. Another option is to modify the sentence to read, "*After he did his homework, he got to play 3/4 of the* amount of time he had already played." Secondly, in *"How much does he play after homework?,"* it is unclear whether Fernando is asking for a unit of time (e.g., hours, minutes) or possibly some number of videogames. To clarify, the question posed could be modified to reference a unit of time. For example, the question could be *"How much* time *does he play after homework*?" or *"How* many hours *does he play after homework*?" These potential modifications are examples of sentence frames (Wisconsin Center for Education Research (WCER) (2014). Fernando's story informed us about how sentence frames such as "How __ of __ of an hour" or "How many hours" could have helped him and other students to tell their stories using fluent sentences that included details (Coleman & Goldenberg, 2009). As we progressed through the four stages of the DAP, we came to recognize the complex language needed to develop a multiplication story. Rather than simply devising questions and addressing keywords in this problem-solving episode, Fernando and other EL students would have benefitted from being given sentence frames that they could have applied directly in their stories.

Discussion and Implications

In this paper, we described what we learned from using the DAP during a problem-solving episode to support Fernando and other EL students to develop their use of the mathematics register in English. Fernando created a multiplication story in response to a task that demonstrates his mathematical understanding of the part-whole notion of fractions as well as the concept of fraction as operator (Charalambous & Pitta-Pantazi, 2007; Lamon, 2007). While Fernando's story is mathematically sophisticated, it was also the case that the story could be improved with the addition of a few key phrases. We observed this during the third stage of the DAP when students were presenting their stories to peers and the entire class.

This example demonstrates how the DAP can be a helpful tool to inform instruction about how to support EL students with the linguistic expectations associated with writing mathematics related stories. Specifically, in this case, how the introduction of words and expressions through the use of sentence frames could support the development of students' English language fluency in the domain of mathematics. Undoubtedly, the demands of writing a multiplication story are linguistically complex (Martiniello, 2008). To address this complexity, an implication for instruction is how through the explicit use of sentence frames (WCER, 2014), we could have helped EL students such as Fernando address this complexity by providing them at the initiation of the problem-solving episode with expressions such as "how much time" and "how many hours" that they could have used in their stories.

After observing Fernando's response and other students' responses to the task, we noted the importance of not only identifying potential questions and key phrases and words needed to support EL students during task implementation, but the value as well of identifying potential language supports for students such as sentence frames that students could have used in their stories. The use of the DAP helped us gain this insight. In addition to providing guidance on how to integrate ESL strategies as students worked through Pólya's (1945/1986) four stages of problem solving, the use of the DAP informed us about how to support students' burgeoning mathematics register to construct fluent and detailed sentences involving time.

References

Abedi, J., & Gándara, P. (2006). Performance on English language learners as a subgroup in large-scale assessment: Interaction of research and policy. *Educational Measurement: Issues and Practices, 25*(4), 36-46.

Castellón, L.B., Burr, L. & Kitchen, R.S. (2011). English language learners' conceptual understanding of fractions: An interactive interview approach as a means to learn with understanding. In Téllez, K., Moschkovich, J.N., & Civil, M. (Eds.) *Latinos and mathematics education: Research on learning and teaching in classrooms and communities* (pp. 259-282). Charlotte, NC: Information Age Publishing.

Charalambous, C., & Pitta-Pantazi, D. (2007, March). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics*, *64*(3), 293- 316.

Coleman, R. & Goldenberg, C. (2009). What does research say about effective practices for English learners? *Kappa Delta Pi Record*, *46*(1), 10-16. doi: 10.1080/00228958.2009.10516683

Cook, L., & Friend, M. (1995). Co-Teaching: guidelines for creating effective practices. *Focus on Exceptional Children*, *28*. https://doi.org/10.17161/fec.v28i3.6852

Creswell, J. W. (2014). *Qualitative inquiry and research design: Choosing among five approaches* (3rd edition). Thousand Oaks, CA: Sage Publications.

Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire*. Clevedon: Multilingual Matters.

Franke, M.L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Reston, VA: National Council of Teachers of Mathematics.

Garrison, L., & Mora, J. K. (2005). Adapting mathematics instruction for English-language learners: The language-concept connection. In L. Ortiz-Franco, N. G. Hernandez, & Y. De La Cruz (Eds.), *Changing the faces of mathematics: Perspectives on Latinos* (pp. 35-48). Reston, VA: National Council of Teachers of Mathematics.

Kitchen, R.S. (2014). Using formative assessment to promote innovative pedagogy for ELLs: Introducing the Discursive Assessment Protocol. In. M. Civil & E. Turner, (Eds.), *The common core state standards in mathematics for English language learners: Grade K-8* (pp.127-138). Alejandra, VA: TESOL.

Kitchen, R. S., Burr, L., & Castellón, L. B. (2010). Cultivating a culturally affirming and empowering learning environment for Latino/a youth through formative assessment. In R. S. Kitchen & E. Silver (Eds.), *Assessing English language learners in mathematics* (Vol. 2, pp. 59–82). Washington, DC: National Education Association.

Kitchen, R. S., DePree, J., Celedón-Pattichis, S. & Brinkerhoff, J. (2007). *Mathematics education at highly effective schools that serve the poor: Strategies for change*. Mahwah, NJ: Lawrence Erlbaum Associates.

Kitchen, R. S. & Wilson, L. D. (2004). Lessons learned from students about assessment and instruction. *Teaching Children Mathematics, 10*(8), 394-399.

Krashen, S. D. (2009). *Principles and practice in second language acquisition*. Oxford: Pergamon.

Kulm, G., Wilson, L. D., & Kitchen, R. S. (2005). Alignment of content and effectiveness of mathematics assessment items. *Educational Assessment, 10*(4), 333-356.

Lamon, S. (2007). Rational numbers and proportional reasoning. In Lester, F. K. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Reston, VA: National Council of Teachers of Mathematics.

Martiniello, M. (2008). Language and performance of English-language learners in math word problems. *Harvard Educational Review, 78*(2), 333-368.
Miles, M. B., Huberman, A. M., & Saldaña, J. (2013). *Qualitative data analysis: A methods sourcebook*. Thousand Oaks, CA: Sage Publications, Inc.
Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, *6*(1), 45-57.
Moschkovich, J. (2015). Academic literacy in mathematics for English learners. *The Journal of Mathematical Behavior*, *40*, 43-62.
Pólya, G. (1986). How to solve it: A new aspect of mathematical method. Princeton University Press. (Original work published in 1945)
Ryve, A. (2011). Discourse research in mathematics education: A critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education, 42*(2), 167-198.
Schleppegrell, M. J. (2004). *The language of schooling: A functional linguistics perspective*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
Schoenfeld, A. (1985). Metacognitive and epistemological issues in mathematical understanding. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 361-380). Hillsdale, NJ: Erlbaum.
Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: McMillan.
Sileo, M., & van Garderen, D. (2010). Creating optimal opportunities to learn mathematics: Blending co-teaching structures with research-based practices. *Teaching Exceptional Children, 42*(3), 14–21.
Silver, E. A., & Smith, M. S. (1996). Building discourse communities in mathematics classrooms: A worthwhile but challenging journey. In P. Elliott (Ed.), *Communication in mathematics, K-12 and beyond* (pp. 20-28) [1996 yearbook of the National Council of Teachers of Mathematics]. Reston, VA: National Council of Teachers of Mathematics.
Valenzuela, A. (Ed.), (2005). *Leaving children behind*. Albany, NY: State University of New York Press.
Wisconsin Center for Education Research (WCER). (2014). 2012 amplification of the English language development standards: Kindergarten – grade 12. WCER: Board of Regents of the University of Wisconsin.
Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development*. Retrieved from Stanford University: http://ell.stanford.edu/content/mathematics-resources-additional-resources.