SURVEY OF PRESERVICE TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE FOR STUDENTS' MULTIPLICATIVE REASONING

Christine K. Austin
Kent State University
causti17@kent.edu

Karl W. Kosko
Kent State University
kkosko1@kent.edu

Multiplication and division are vital topics in upper level elementary school. A teacher's pedagogical content knowledge (PCK) influences both instruction and students' learning. However, there is currently little research examining teachers' PCK within this domain, particularly regarding professional education of future teachers. To help address this need, the present paper presents an initial validity argument for a survey of preservice teacher's PCK for multiplication and division.

Keywords: Teacher Knowledge; Number Concepts and Operations

Overview & Purpose

Multiplicative reasoning is a critical concept in upper elementary school (grades 3-5) that facilitates student reasoning of later mathematics concepts (Hackenberg, 2010). Whole number multiplication and division is formally introduced in grades 3-5 (CCSSI, 2010), leading to their inclusion in initial licensure mathematics methods courses for early childhood, elementary, and middle grades preservice teachers (PSTs). However, there is limited research on PSTs' professional knowledge in this area (Thanheiser et al., 2014). Such literature tends to focus on PSTs' understanding of the content (Harkness & Thomas, 2008; Menon, 2003), and often conveys a large portion of novice teachers lack sufficient understanding of declarative knowledge related to multiplication and division. Yet, the professional knowledge needed to teach mathematics, or Mathematical Knowledge for Teaching (MKT), involves more than a deep understanding of the content (Hill et al., 2008b). Pedagogical Content Knowledge (PCK) "goes beyond knowledge of subject matter," in that it is a "particular form of content knowledge that embodies the aspects of content most germane to its teachability" (Shulman, 1986, p. 9). Indeed, there is evidence to suggest that PCK for mathematics is more sophisticated than content knowledge (Copur-Gencturk et al., 2019), but there is relatively little study of PSTs' PCK for multiplication and division of whole numbers (Thanheiser et al., 2014). One reason for this is the relative difficulty in defining and creating measures of PCK (Copur-Gencturk et al., 2019; Hill et al., 2008a). In our own work, we sought such a measure to gauge the effect of a teacher education initiative. The lack of a measure of PSTs' PCK for multiplication and division, therefore, fueled our need to create such a measure. Thus, the purpose of this study is to construct an initial validity argument for a survey of preservice teachers' pedagogical content knowledge for elementary children's multiplicative reasoning.

Background Literature & Theoretical Perspectives

Pedagogical Content Knowledge

This study reports on the design and initial validation of an MKT assessment of whole number multiplication and division. Current assessments of MKT have focused on either specific courses, such as Geometry or Algebra I (Herbst & Kosko, 2014; McCrory et al., 2012) or a wide range of content within a single mathematical domain, such as numbers and operations (Hill et al., 2008a). For example, McCroy et al. (2012) developed an instrument to test teachers' mathematics-teaching-knowledge of Algebra, constructing items specific to student reasoning of algebra problems. McCrory et al.'s (2012) definition of mathematics-teaching-knowledge is similar to PCK, as it includes knowing a student's mathematical reasoning and understanding possible misconceptions. Similar to McCrory et al. (2012), Herbst and Kosko (2014) developed items to investigate MKT in Geometry teachers by constructing items based on students reasoning and approach to geometry problems. Hill et al. (2008a) also created items focusing on PCK, but the majority of their assessment is focused on both common and specialized content knowledge for teaching. While the aforementioned efforts for designing PCK items have met some success, when scholars have designed MKT assessments for specific concepts, such as fractions, the focus tends to be on content knowledge, and not PCK (Izsák et al., 2019).

Although prior research provides useful contributions to the field, the lack of specified focus on aspects of PCK in measurement development has led to underspecification of the domain both within and beyond our focus on multiplication and division. Analyzing items from two different MKT measures, Copur-Cencturk et al. (2019) note that "what constitutes PCK and how PCK differs from [specialized content knowledge] SCK are not well articulated… We need a more in-depth understanding of teachers' instructional strategies that help their students learn and how teachers' knowledge of students' thinking is revealed in mathematics instruction and informs their teaching" (p. 494). Hill et al. (2008a) suggest the problem is two-fold in that there is a lack of research on teachers' PCK and that "the field has not developed, validated, and published measures to assess" (p. 373) such knowledge. Since Hill et al.'s (2008a) writing this statement, items assessing PCK have been successfully written and validated. However, these are typically couched in an overarching assessment of MKT (Depaepe et al., 2015; Herbst & Kosko, 2014). By contrast, this paper focuses explicitly on PCK. Shulman (1986) defines aspects of PCK as:

> An understanding of what makes the learning of specific topics easy or difficult: the conception and preconceptions that students of different ages and backgrounds bring with them to the learning of … frequently taught topics and lessons. If those preconceptions are misconceptions … teachers need knowledge of the strategies ... in recognizing the understanding of learners (p. 9).

There are two primary subdomains of PCK: Knowledge of content and students (KCS) and knowledge of content and teaching (KCT). KCS is defined by Ball et al. (2008) as the knowledge of knowing students as well as knowing the mathematical framework. Within this domain of PCK it is required that teachers know how a student is going to think through a problem and anticipate what problems students will find daunting and confusing (Ball et al., 2008). In contrast, Ball et al. (2008) defined KCT as having the knowledge of how to effectively teach combined with the knowledge of the mathematical subject matter. Teachers with a high level of KCT can use various models to illustrate a concept to students at varying stages of learning (Ball et al, 2008).

Both subdomains have been successfully assessed within the literature. Hill et al. (2008a) developed an assessment to identify KCS and was, to an extent, successful. The findings suggest that in order to answer an item pertaining to a common student error, student understanding, common student developmental sequences, and common student computations a teacher must possess content knowledge (CK) and KCS (Hill et al., 2008a). McCroy et al. (2012) suggested a framework to develop an assessment to measure KCT outside of Algebra by establishing the difference of math knowledge and teaching knowledge. In addition, Herbst and Kosko (2014) constructed an instrument to measure KCS and KCT as well as common content knowledge (CCK) and specialized content knowledge (SCK). The items developed to assess KCS in teachers "probe[d] for their knowledge of students' conceptions and errors in tasks" pertaining to geometry (Herbst & Kosko, 2014, p. 41). Their assessment was able to detect that experienced teachers were more successful at identifying student conceptions/misconceptions than less experienced teachers. The KCT items Herbst and Kosko (2014) constructed followed the same trend; experienced teachers were better able to determine appropriate tasks and examples to effectively illustrate a concept in comparison to less experienced teachers (Herbst & Kosko, 2014). These prior efforts at constructing PCK items, in the context of MKT as a larger construct, informed our own efforts at item design.

The present paper focuses on KCS of Multiplication and Division and follows item design recommendations of Ball et al. (2008) and Herbst and Kosko (2014). Specifically, items were written to assess teachers' knowledge of students' conceptions and errors in whole number multiplication and division for grades 3 to 5. In the next section, we describe this process in detail.

Development of the PCK-MaD Assessment

In this section, we describe the development of items for an assessment of Pedagogical Content Knowledge for Multiplication and Division (PCK-MaD). Items for the initial version of PCK-MaD were designed specifically to assess the KCS dimension of Ball et al.'s (2008) MKT framework. We anticipate including KCT items in a later version of the assessment but sought to focus on KCS as an initial step. Following recommendations from prior work in this area (Ball et al., 2008; Herbst & Kosko, 2014), we designed items focusing specifically on variations in upper elementary school children's conceptions of multiplication and division. To do this, we focused on grades 3-5 Common Core Standards for Mathematics on multiplication and division standards (CCSSI, 2010) as a means of identifying key concepts to write items. Next, we conducted a literature review of mathematics education research on these and related concepts that described the nature of children's reasoning. We paired this review of research with a review of practitioner resources (Battista, 2012; Van de Walle et al., 2019).

Figure 1 provides an example item to help illustrate this process of item design, writing, and revision. The item, designated M01, was designed to assess teachers' knowledge of children's developmental skip-counting, and aligns with CCSS standard 3.0A.A.1 specifying that children need to interpret products of whole numbers. Variations of skip-counting have been observed by researchers, including a phenomenon where students begin to miss certain skip-counts (Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005). Steffe (1994) describes this as a point where children are beginning to compose iterable units, counting with whole numbers other than 1, but that this action is still very dynamic for the child. Rather, the composite whole number has not been fully abstracted for the child, and as they attempt to skip-count, they may lose track between coordinating the unit to be skip-counted and coordinating the number of skip-counts. In Figure 1, we illustrate this form of reasoning with a context of multiplying 7 and 8, and an illustration of skip-counting with one's fingers. Distractors were included to represent other points in learning progressions described for practitioners (Battista, 2012; Van de Walle et al., 2019). For example, Battista (2019) describes repeated addition as distinct from uncoordinated skip-counting. It is also a distinction that may be difficult for some PSTs to observe, making option #4 a useful distractor. Although Figure 1 provides a final version of item M01, multiple revisions occurred as language and figures were reviewed and critiqued by the project team.

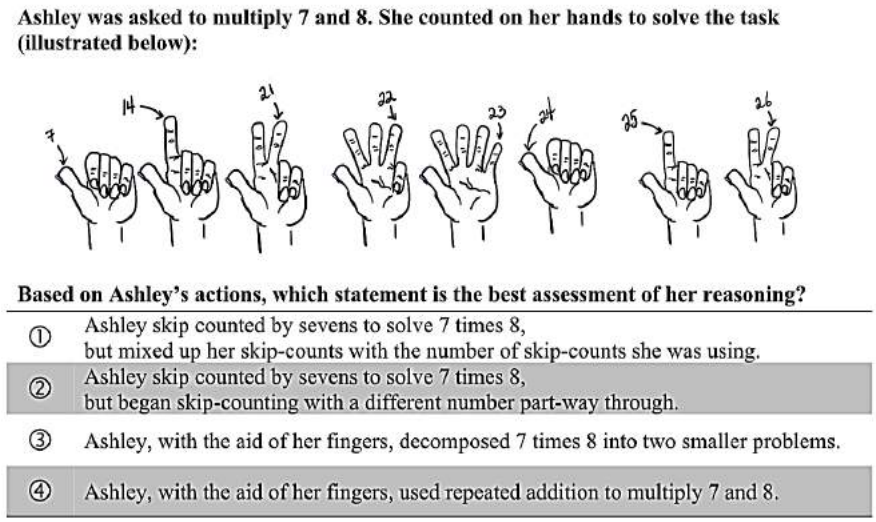


Figure 1. Example PCK-MaD item assessing for understanding of children's developmental skip-counting.

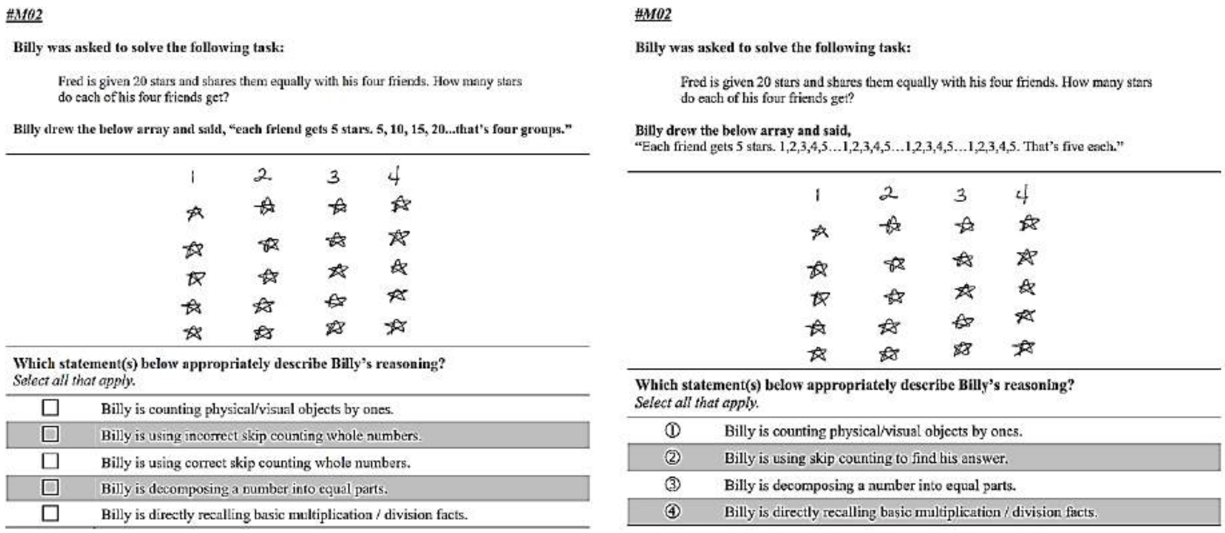
After all items had been successfully vetted by project team members, we conducted cognitive interviews with two elementary math coaches who were widely recognized by the field for their expertise. Cognitive interviewing is a process in which a participant engages in a one-on-one interview to complete an assessment. After completing each item, the participant is asked what they thought the item was asking them to do, what they answered, and why they answered the way they did (Karabenick et al., 2007). For the PCK-MaD, cognitive interview data was used to examine whether items were interpreted as intended, and whether rationales for responses focused on aspects of students' mathematical thinking (i.e., KCS). Each interview was roughly 2 hours in length, but within this time through feedback on items was given.

Item M15, Figure 3, illustrates an example of a question that was *not* altered based on the feedback from the two math coaches. By contrast, Figure 2 depicts an example that was drastically modified due to the constructive criticism. This item was originally designed to be multiple response but was modified to become a multiple-choice item. In addition, the language of the item pertaining to the sample students reasoning was revised to be clearer of the intended thought process due to the discrepancy of responses from the two expert teachers:

Expert Teacher #1: "Billy is decomposing items into equal parts."

Expert Teacher #2: "It looks like he is counting visual items by one"
In response to why she answered that way:
Expert Teacher #2: "Because of the fact that all twenty were represented by stars so it looks like he counted 1, 2, 3, 4, 5."

The different responses illustrated to us that the item stem was unclear, and some of the options may have been interpreted in ways we did not intend. The reasoning for "Billy" was modified to be clearer of his mathematical reasoning of counting the stars and changed to a multiple-choice item to not distract the users further. Unfortunately, the cognitive interviews also resulted in one item being removed completely from the assessment due to the overall confusion of the participants. Some items were revised with very minor adjustments (missing punctuation, a typo in an image, etc.). Following cognitive interviews, 9 items received some revision (minor to moderate), 4 items remained as-is, and 1 item was fully removed.



Original Question Revised Question

***Figure 2*: PCK-MaD questions M02 example of revision after cognitive interviews.**

Another outcome of our cognitive interviews was a realization of the cognitive demand of several items. Evidence from the literature suggests that KCS items may be more difficult than other MKT domains (Copur-Cencturk et al., 2019; Herbst & Kosko, 2014), and we found that many of our KCS items were indeed more difficult. Therefore, we created three additional items, following cognitive interviews, in an effort to have KCS items with an easier difficulty level.

Following the framework of Herbst and Kosko (2014) items were developed to measure KCS in pre-service teachers. Revision of the items based on the feedback from the expert teachers resulted in the pilot PCK-MaD assessment. The adjustments made to the items added to clarity and refined the level of difficulty of the language. However, to properly vet the items and gather further validity of the assessment, we collected pilot data from preservice teachers (PSTs) enrolled in a teacher education program. This process served to collect validity evidence for an initial validity argument for the PCK-MaD.

Method

Sample and Procedure

Participants included 58 PSTs, with 47 preparing to become elementary teachers (grades K-3 with an endorsement option for grades 4-5) and 11 preparing to become middle grades teachers (grades 4-9). Participants were in the latter half of their teacher education (31 juniors; 27 seniors). The majority of junior participants were elementary PSTs (n=27) preparing to take the first of two mathematics methods courses. Although these participants had some pedagogical coursework and field experience, they hadn't received formal education on PCK for multiplication/division. Four juniors were middle grades PSTs who had completed the first of two mathematics methods courses. Senior elementary PSTs (n=20) had completed two mathematics methods courses, with several field-based assignments relating to multiplicative reasoning across grades K-3. All participating elementary PSTs expressed their intent to complete an additional mathematics methods course focusing on grades 4-5, but none had completed this course at time of data collection. The majority of middle grades PSTs (7 of 11) were seniors and were enrolled in the second of two mathematics methods courses in their program.

Analysis and Results

Given the early stage of developing our PCK survey, the present paper examines validity evidence from test content and response processes. Validity evidence for *response processes* refers to "whether test takers are, in fact, reasoning about the material given instead of following a standard algorithm applicable only to the specific items on the test" (AERA et al., 2004, p. 15). Wolf and Smith (2007) suggest that psychometric measures can be used to assess the degree that the theoretical rationales for item content align with response processes. Therefore, to examine evidence for response processes in the present paper, we conducted a classical item analysis to examine the internal reliability of items and the resulting measure, and to examine the relative difficulty of those items in comparison with one another.

The PCK survey included 15 questions, with six questions conveyed in a multiple-response (i.e., select all that apply) format. For example, question M15 presents six different student algorithms and asks the survey respondent to select those that used all partial products (see Figure 3). This effectively conveys six different items for M15. Thus, for the 15 questions we examined, there were 41 items, due to the six multiple response questions. Our initial item analysis model, including all 41 items, resulted in a Cronbach's alpha coefficient of .47. For surveys and piloted assessments such as the one in this paper, the typically accepted threshold is at or near *.70* (Nunnally & Bernstein, 1992). Therefore, we examined the point-biserial correlations for each item to identify candidates for removal. Point-biserial coefficients below .30 are considered to not meaningfully contribute to the total score, possibly due to variance in response (Crocker & Algina, 2006). Rather than remove all such items, it is customary to remove one item at a time, so that the remaining items' point-biserial coefficients can be recalculated for a new model. In addition to identifying particularly low coefficients, items are examined in the context of their theoretical contributions to the model, as well as evidence from cognitive interviews and/or written work on the surveys. For example, the sixth item on question M15 had an initial point-biserial coefficient of .021 (see Figure 3). The low coefficient essentially flagged the item for review. We then considered evidence from our cognitive interviews in which unfamiliarity with the lattice method and how it functioned mathematically resulted in incorrect responses. Thus, this option for question M15 was removed. A similar process took place for all iterations of item analysis. Our final model included 21 items, from nine questions, with a Cronbach's alpha coefficient of .68. This suggests at least 68% of the variance in responses is due to the measured construct (PCK for multiplication and division). Point-biserial coefficients for most items were above or near the .30 threshold. Item difficulty for the remaining items ranged from .20 (20% of the sample answered correctly) to .90 (90% of the sample answered correctly), with a mean score of 14.88 (SD = 3.14, Range = 5 to 16).

Validity evidence for *test content* considers how well assessment content represents PCK for children's multiplicative reasoning, and how well this content aligns with interpreting PSTs' scores (AERA et al., 2004). To analyze this, we will examine the intended purpose of the assessment (i.e., to measure the effect of teacher education) using an independent samples t-test for PCK scores of juniors and seniors. Results were statistically significant (t = 2.686188, df = 56, p = .00933), indicated that senior PSTs had higher PCK scores (15.8674) than junior PSTs (13.6690). To ensure the comparison between junior and senior PSTs was not influenced by major, we examined the difference between elementary and middle grades PSTs' scores and found no statistically significant difference (t = 0.673, df = 56, p = .503). We also examined whether PSTs who had a field placement in grades 3-5 would have higher PCK scores and found no statistically significant difference between those with and without such field experience (t = 0.396, df = 56, p = .743). Considered collectively, these results suggest that, for participants in the current study, the PCK survey distinguishes between PSTs who are earlier or later in their teacher education program. Such a difference does not appear to be due to intended licensure (elementary or middle grades) or having grades 3-5 field experience.

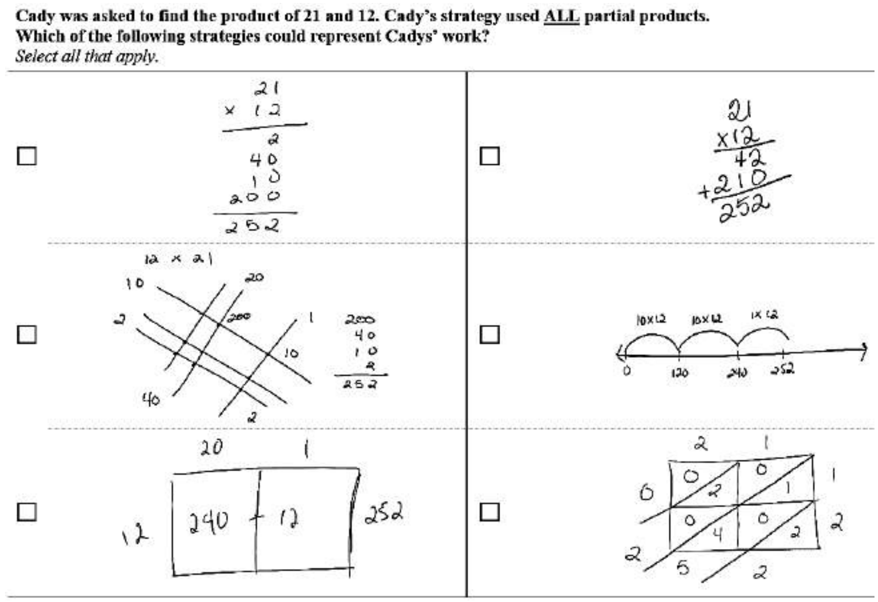


***Figure 3*: PCK-MaD that assesses children's strategy-use with partial products.**

Discussion

To our knowledge, there is no prior assessment for PCK of multiplication and division for whole numbers. This study reported on the initial piloting of our assessment of PST's knowledge of content and students (KCS) with a focus on multiplicative reasoning. Given the need for additional research on PSTs' PCK for multiplication and division (Thanheiser et al., 2014), development of a measure for this domain has the potential for informing the field in this regard. The findings of this study suggest that our survey can distinguish between PCK scores of PSTs at different levels of teacher education (i.e., senior vs. junior). On one hand, this provides useful validity evidence for the PCK-MaD's ability to distinguish between PSTs at different points in their teacher education. However, this finding also lends support for the effectiveness of teacher education programs at developing PSTs' PCK. Both implications of this particular finding, while useful, should be interpreted with caution as the current study represents an initial pilot of an assessment and involves a sample from a particular teacher education program.

Psychometric data from the PCK-MaD item analysis and data from the two cognitive interviews suggest that the piloted assessment does measure the intended construct. However, future research is needed to improve the assessment. Results suggest initial support for an assessment of KCS, but additional items focusing on KCT should be developed. Further, results here focus predominately on responses from PSTs, suggesting a need to examine responses from inservice teachers to establish a better understanding of normative KCS in the field. Despite the early stage of this work, results suggest that the PCK-MaD may be used as-is for assessing the effect of teacher education initiatives.

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