PROSPECTIVE K-8 TEACHERS' PROBLEM POSING: INTERPRETATIONS OF TASKS THAT PROMOTE MATHEMATICAL ARGUMENTATION

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This study examines pre-service teachers' (PSTs') views of tasks that engage students in mathematical argumentation. Data were collected in two different mathematics courses for elementary school education majors (n = 51 total PSTs). Analyzed were (a) written journals in which PSTs defined tasks that promote student engagement in argumentation, (b) tasks PSTs posed to engage students in mathematical argumentation, and (c) accompanying explanations in which PSTs motivated tasks they posed. The analysis revealed that PSTs interpret tasks that foster argumentation in terms of activities of argumentation that a task elicits and space for argumentation that the task provides. Several features that PSTs associated with each of the two major task characteristics were identified. While posing tasks to engage students in argumentation, PSTs did not place equal emphasis on all of the identified features.

Keywords: Reasoning and Proof, Mathematical Knowledge for Teaching, Teacher Education-Pre-service.

Background

Curricular standards in mathematics recognize mathematical argumentation as an essential disciplinary practice with which all students should engage and become proficient (e.g., National Governors Association Center for Best Practices and Council of Chief State School Officers [CCSSM], 2010; National Council of Teachers of Mathematics [NCTM], 2000). Engaging students in constructing viable arguments and critiquing the reasoning of others is the desired instructional goal (CCSSM, 2010). Past research on argumentation shows, however, that contrary to curricular recommendations and recognized importance of argumentation in student learning, teaching mathematics with a focus on argumentation is still far from a common practice (e.g., Bieda 2010; Staples, Bartlo, Thanhaiser, 2012).

Past research with practicing (and pre-service teachers, PSTs) documented challenges that teachers face while facilitating argumentation in their classrooms (e.g., Bieda 2010; Kosko, Rougee, & Herbst, 2014), explored the role of the teacher in promoting argumentation (e.g., Aylon & Even, 2016; Conner, Singlertary, Smith, Wagner, Francisco, 2014; Graham & Lesseig, 2018), and explored how teachers interpret argumentation in the context of mathematics classrooms (e.g., Park & Magiera, 2019). While overall, the research interest in argumentation is growing, research attention to teachers' views of tasks that promote student engagement in mathematical argumentation has been limited.

Researchers agree that tasks play an essential role in how students experience mathematics (Krainer, 1993; Simon & Tzur, 2004; Zaslavsky, 2008). However, research on curricular materials reveals that school mathematics textbooks, even textbooks designed to support mathematics curriculum reforms, offer limited collections of tasks that, by their inherent design, provide opportunities for engaging students in argumentation (e.g., Bieda, Ji, Drwencke, & Pickard, 2013; Dolev & Even; 2015; Stacey & Vincent, 2009). Understanding how teachers interpret tasks that engage students in argumentation could help gauge students' opportunities for experiencing argumentation in mathematics classrooms. Research-based information about teachers' views of tasks that engage students in mathematical argumentation can also aid the efforts of helping teachers develop a more comprehensive knowledge

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of argumentation. Thus, in a bid to address these existing gaps, this study uses problem-posing as a context for exploring elementary PSTs' views of mathematical tasks that engage students in the practice of argumentation. This research was guided by the following question: *What characteristics of tasks that promote mathematical argumentation emerge from the analysis of problems PSTs' pose to build students' capacities in mathematical argumentation, and PSTs' descriptions of problems that engage students in argumentation?*

Conceptual Framework

Problem Posing

Problem-posing, which includes designing new and modifying existing tasks, is recognized as an essential element of mathematical activity (Silver, 1994). Teachers' ability to design and pose mathematical tasks is one of the central aspects of mathematics teaching (Krainer, 1993; NCTM, 2000). Classroom problems provide students with the opportunity for thinking and learning (Smith & Stein, 1998). Thus problem-posing is perceived as integral to teaching a "high leverage" practice, a gateway to understanding that serves as a learning and instructional tool (Ball & Forzani, 2009). While posing problems, teachers go beyond thinking about problem-solution, they need to consider the overall goal of the task, think about what and how students can make sense of the mathematics they learn, and what understandings, skills, and attitudes they develop (Crespo, 2015; Lavy & Shriki, 2007; NCTM, 1991). Researchers recognize that the activity of problem-posing can provide a window into an understanding of teachers' mathematical and pedagogical content knowledge (Ellerton 2015; Lee, Capraro, & Capraro, 2018).

Mathematical Argumentation

Toulmin, Rieke, and Janik (1984) described argumentation broadly as "the whole activity of making claims, challenging them, backing them up by producing reasons, criticizing those reasons, rebutting those criticisms, and so on" (p. 14). This description is consistent with the notion of argumentation presented in the Standard for Mathematical Practice #3 (CCSSM, 2010). Mathematics education researchers generally agree that in school mathematics, argumentation involves a wide range of activities. These activities include constructing, validating, or refuting mathematical claims, producing and criticizing justifications, formulating conjectures, generalizing, representing mathematical ideas, constructing counterexamples, or communicating reasons, to name some. (e.g., Lakatos, 1976; Knudsen, Lara-Meloy, Stevens, & Rutstein, 2014; Krummheuer, 1995; Ramsey & Langrall, 2016). The existing frameworks that guide the examination of textbook tasks for their affordances of engaging students in argumentation (e.g., Bieda et al., 2014; Stylianides, 2009) classify the kinds of argumentation-related activities elicited by the task. Given that the focus of this research was on PSTs' interpretations of tasks that engage students in argumentation, not on the implementation of classroom tasks to engage students in argumentation, Toulmin et al. (1984) broad description of argumentation together with frameworks proposed to classify the types of argumentation-related activities elicited by written tasks provided an attractive guide for this study. They allowed negotiating a wide range of meanings that PSTs bring while thinking about tasks that engage students in argumentation and to place argumentation within the individual and social space a task might create for student engagement in argumentation.

Method

Participants and Study Context

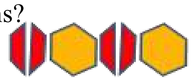
The study was conducted in the Midwestern university in the U.S. Participants were 51 PSTs preparing to teach grades 1-8 mathematics enrolled in two mathematics content and concurrent pedagogy with field experience set of courses for elementary education majors. The two pairs of courses were *Number Systems and Operations for Elementary School Teachers* and *Teaching Elementary School Mathematics* (n = 23) and *Algebra and Geometry for Teachers* and *Teaching Middle School Mathematics* (n = 28). Curricula of each set of courses were coordinated. In the context of their mathematics courses, PSTs studied concepts fundamental to the K-8 mathematics and engaged in mathematical argumentation as learners. In their corresponding pedagogy with field experience courses, they focused on teaching strategies that support students learning of K-8 mathematics and support students' mathematical reasoning skills. They conducted focused observations in their field placement classrooms to identify teacher moves, instructional strategies, and classroom interactions that supported student reasoning. In the context of their education and field experience work PSTs also prepared and conducted two problem-based interviews with students for the purpose of engaging students in the practice of mathematical argumentation and learning about student mathematical thinking.

Data and Data Analysis

Collected in the Number Systems and Operations for Elementary School Teachers course were (a) written journals in which each PST described tasks that engage students in argumentation, (b) two tasks each PST posed (one at a time) in preparation for their interviews of elementary school students, and (c) explanations in which each PST described why task they posed creates an environment for student engagement in argumentation.

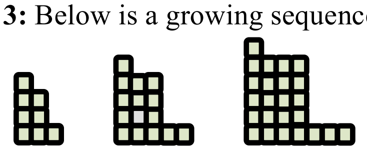
Collected in the Algebra and Geometry for Teachers course were: (a) written journals in which each PSTs described tasks that engage students in argumentation, (b) PSTs' analyses and critiques of three instructor-provided tasks (completed one at a time) in which they discussed each task's potential to engage students in argumentation, and (c) revisions of instructor-provided tasks PSTs' proposed to enhance each task's potential to engage students in argumentation and PSTs' explanations for each revision. Instructor-provided tasks that provided context for PSTs' problem-posing activity are shown in Figure 1.

Task 1: How many red pattern blocks will be used if the pattern of figures is extended until there is a total of 13 polygons?



Task 2: Kay made summer lemonade from a mix using l2 tablespoons of lemonade mix and 20 cups of water. How many tablespoons of lemonade mix will she need if she plans to use 30 cups of water to make lemonade that tastes just the same?

Task 3: Below is a growing sequence of figures.



a) Draw the 1st, 5th, and 6th figures
b) How is the pattern changing?
c) What would the 100th figure look like? How many tiles it has? How can you justify your prediction?

Figure 1: Instructor-provided Tasks that PSTs Analyzed and Revised

Qualitative analytical-inductive methods were used for data analysis. In the first round of the analysis, a large subset of all data (about 25%) that consisted of all types of artifacts was first carefully annotated to discern PSTs' perceptions of tasks that engage students in argumentation. The goal was to create a code-book that could reliably capture task characteristics identified across PSTs' definitions, tasks they posed (designed or revised), journals in which they described why tasks they posed could engage students in argumentation, or interviews during which they discussed their tasks. To illustrate the coding process, consider included in Figure 2 task that PST A44 posed together with explanation PST A44 provided about her task. Following the initial round of the analysis, the annotations were further compared and contrasted, and their descriptions revised to formulate a final set of codes that were then applied to the entire corpus of data. If any new task characteristics were identified during this process, new codes were introduced, and the code-book was augmented. The final list of codes was further compared and contrasted until no new task characteristics were identified. In the final stage of analysis, all codes were again compared and contrasted, leading to the identification of two major task characteristics. Task features identified across each PST's responses were then tabulated to identify the overall patters in PSTs' interpretations of tasks that facilitate student engagement in mathematical argumentation.

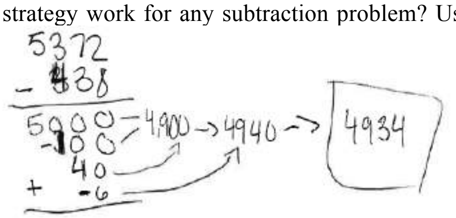
PST A44's Task 1 (task posed)	Annotations
Kim has 5,372 songs on iTunes. She deletes 438 songs. How many songs does she have on iTunes now? A student solves this problem by subtracting the thousands, hundreds, tens, and one's from 5,372 by 438. Then she adds the sum of each number place together to get the final answer of 4,934 songs. Does this strategy work? Why or why not? Can this strategy work for any subtraction problem? Use examples to explain your reasoning. 	Call for justification of a strategy (Does this strategy work? Why or why not?) Call for evaluating reasoning of others Call for generalization (Can this strategy work for any subtraction problem?) Call for communicating thinking, reasoning (Use examples to explain your reasoning)
PST A44's Explanation about Task 1	Annotations
This task takes a simple subtraction problem and turns it into an engaging mathematical problem for the student. The student must understand how this subtraction problem was solved by someone else using what I assume will be a method that is different from how the student traditionally solves a subtraction problem. The task asks several questions of the student to clarify that they understand how the problem was solved and help them build upon their argumentation of how they got the solution and how it works. The task can be applied to other example problems to help the student with their understanding of the subtraction strategy and to help them explain their mathematical reasoning. I think the most likely problem that could occur is that the student may not fully grasp or understand the technique given to solve the subtraction problem. They may not get the idea of subtracting the place value or may struggle with the negative numbers that show up in this problem because they are not used to working with negatives in a standard subtraction problem. Lastly, it might be the wording of the problem, that stumps the student. However, with a little help, I do think that most sixth graders can absolutely understand this technique and argue how it works. The main math skill required to understand this task is place value and sixth graders should certainly have a strong understanding of this concept.	Task is engaging Task engages in evaluating the reasoning of others Task engages in generalizing Task is non-routine, requires deeper thinking, challenging Task builds on student existing knowledge, understanding

Figure 2: Example of Task Posed and Task Explanation (PST A44, Task 1)

Results

Table 1 summarizes the characteristics of tasks identified across PSTs' responses. The analysis revealed that while posing tasks to engage students in argumentation, PSTs considered (a) activities of argumentation in which students could engage given their task and (b) space for argumentation that their task provides. Overall across the analyzed tasks, task explanations, and PSTs' definitions of tasks that engage students in argumentation, individual PSTs included between 2 to 11 different task features.

Table 1: PSTs' views about tasks that promote mathematical argumentation

Major Task Characteristics	Task Features	(n, %*)
Activities of Argumentation	Task promotes providing justifications	48 (94%)
	Task encourages making generalizations	34 (67%)
	Task elicits explorations, generating conjectures, evidence, and claims	31 (61%)
	Task promotes evaluating arguments or reasoning of others	28 (55%)
Space for Argumentation	Task elicits communicating thinking and reasoning	47 (92%)
	Task enables the use of multiple solution strategies and ways of thinking	36 (71 %)
	Task draws on students' existing knowledge and allows them to make connections	34 (67%)
	Task engages in deeper thinking, is complex	33 (65%)
	Task supports the use of multiple representations, manipulative materials to guide thinking	31 (61%)
	Task requires that students reflect and make sense of their results	20 (39%)
	Task fosters the development of concepts	14 (27%)

* Rounded to the nearest %.

Activities of argumentation. As summarized in Table 1, PSTs associated argumentation with a broad range of activities in which a student could engage in the context of posed tasks. These activities, categorized as task features, were emphasized across the analyzed data to a different degree. For example, while almost all PSTs (94%), in at least one of their tasks, included an explicit call for justifying a result, claim or strategy, only about half of the participants (55%) designed tasks that would engage students in evaluating arguments or reasoning of others. About two-thirds of PSTs (67%) formulated tasks that engaged students in generalizing, and a little less than two-thirds of PSTs (61%) formulated tasks that encouraged explorations, generating conjectures, evidence, and claims.

Consider the presented earlier task posed by PST A44 (Figure 2). By its design, this task engages a student in evaluating the validity of a given strategy. The task statement requires that a student justifies his or her strategy assessment. The task also elicits thinking about the strategy generality by prompting the student to reason about whether or not the presented strategy can be applied to other subtraction problems. Consider also the following task which PST A15 posed modifying the instructor-provided Task 1 (Figure 1): "How many red pattern blocks will be used if the pattern is extended until there are 200 polygons? Justify your response." PST A15's task also includes an explicit call for justification. The intention to engage a student in generalizing was evident is explanation PST A15 included. PST A15 described her thinking about this task modification and her desire to engage a student in thinking about pattern generalization and exploring and developing a general conjecture about the pattern sharing:

> I increased the number of polygons in order to prevent the student from merely counting the blocks. Increasing the number requires that the student finds a general rule or equation. They can investigate the relationship between the blocks. I also asked the student to justify it. (PST A15, Task 1)

Space for argumentation. This category of task characteristics describes ways in which the PSTs thought about and designed their tasks to create an environment for student engagement in argumentation. As summarized in Table1, PSTs varied in approaches they used to generate a context for student engagement in argumentation within their tasks. For example, almost all PSTs (92%) designed tasks, so the task elicited communicating thinking and reasoning. A large group of PSTs (71%) described and proposed tasks that were open to diverse ways of thinking and solution strategies. The focus on the latter task feature is illustrated with the excerpts below:

> Mathematical tasks that foster argumentation must be challenging., directional, often have more than one specific answer and can be represented in multiple ways (PST A44)

> Flexibility is also very important in fostering mathematical argumentation. Understanding that there are multiple ways to view a problem or multiple solutions that could be found is important because it allows students to challenge ideas and use evidence to prove why their answer is efficient. (PST B17)

About a two-thirds of PSTs, (67%), thought about opportunities their task might give students for drawing on students' existing knowledge and making connections, for engaging in deeper thinking (65%), or for supporting student thinking by encouraging them to use multiple representations or manipulative materials (61%). For example, PST A23 shared:

> Using manipulatives in tasks helps students reason and make claims. When students are able to visualize and make structure of their work, they better understand the problem and are able to make and justify claims. (PST A23)

In contrast, only 39% of PSTs considered tasks that require students to reflect and make sense of their results as one that can engage students in argumentation, and only 27% of PSTs envisioned that tasks that facilitate concept development might engage students in argumentation. An excerpt from PST A18's journal presented below illustrates the former task feature:

> Tasks should encourage students to go back to their own work. A student should see if they used evidence or showed enough work to support their explanation. Are the equations and tables labeled? Can everything be proved? Tasks [that encourage reflection] can help and improve students' skills in making, justifying, and evaluating mathematical claims. (PST A18)

Presented below task posed by PST A2, together with accompanying task explanation, serve as an illustration of PSTs' thinking about how a task that provides space for concept development can engage students in argumentation. PST A2 shared:

> The mathematical task I designed is a word problem that will require students to think about multiplication- this task is designed for a third-grade student. Example task: Sue invited 8 friends to her birthday party. She was making goodie bags for each of her friends. If she puts 5 pieces of candy in each bag how many total pieces of candy does she need? Justify your answer.

While motivating her task PST A2 wrote:

> Mathematical argumentation requires a student to not only explain how they arrived at their answer but to think about the mathematical ideas, concepts, theories, and reasoning that are used in the problem. This task presents students with a fundamental multiplication property – equal-sized groups and repeated addition. Multiplication can be viewed as repeated addition of equal-sized groups. With this task, the student will be exposed to this idea because the task is asking them to find the total pieces of candy when there are 8 groups (bags of candy) with 5 pieces each (candy pieces). […] One potential error I could see with this problem is a student potentially grouping 8 pieces of candy 5 times. If this were to happen, this would still result in the correct solution of 40 and would be a great opportunity to talk about the commutative property of multiplication.

Summary and Discussion

This study contributes to the research on argumentation in school mathematics in two ways. First, it extends the existing analytical models for engagement in mathematical argumentation, which emphasize the structural or cognitive aspects of argumentation (e.g. types of arguments being generated, or degree of justification) and provide a framework that can serve as a guide for the design of tasks that support student engagement in argumentation. Second, it offers insights into PSTs' understanding of mathematical argumentation by describing the kinds of opportunities for argumentation that PSTs envision as they pose tasks to engage students in argumentation.

The analysis revealed that while posing written tasks and thinking about task affordances for engaging students in argumentation, PST considered (a) activities of argumentation in which a student might be involved while working on the task and (b) the space for argumentation that the task generates. PSTs viewed both of these task characteristics as contributing to the overall potential of the task for building students' capacities in argumentation. This finding extends previous conceptual frameworks for analyzing the potential of written tasks for engaging students in argumentation, which exclusively focused on the types of activities of argumentation that task elicits (Bieda et al., 2014; Stacey & Vincent, 2009; Stylianides, 2009).

The results also document task features related to the two identified major task characteristics and show that PSTs do not equally emphasize these features while designing tasks to engage students in argumentation. For example, concerning the activities of argumentation, almost all PSTs in this study posed tasks that elicited justifying. A large proportion of PSTs formulated tasks that promoted conjecturing and generalizing, but tasks that engaged students in evaluating arguments and reasoning of others were posed less frequently. PSTs' choices of task features identified as representative of the space of argumentation posed tasks afforded also varied. For example, the results suggest that PSTs might be more likely to associate opportunities for mathematical argumentation with tasks that elicit communicating thinking and reasoning, or tasks that allow for divergent ways of thinking and solution strategies. About two-thirds of PSTs in this study considered also task complexity, the extent to which task allows students to build on their prior knowledge and make connections, or facilitates the use of multiple representations as a viable task environment that offers space for engaging students in argumentation. Less frequently, PSTs envisioned that tasks that promote concept development or elicit students' reflections on their thinking might provide space for argumentation.

The results of this study provide important insights for mathematics teacher educators about supporting PSTs' visions of argumentation in mathematics classrooms. For example, it is likely that without intentional efforts focused on heightening PSTs' awareness of tasks that engage students in analyzing and critiquing the reasoning of others, PSTs might limit students' opportunities for experiencing this aspect of argumentation. Particularly, because, as reported by Bieda and colleges (2014) in their review of several elementary school textbooks in the U.S., tasks designed to engage students in evaluating claims were rarely present within the elementary school textbooks. This study did not examine how PSTs' envision classroom implementation of tasks for the purpose of engaging students in argumentation. To generate a more robust picture of PSTs' knowledge in the area of mathematical argumentation future research should investigate PSTs' interpretations of tasks that engage students in argumentation and the nature of opportunities for engaging students in argumentation PSTs see in classroom tasks, with concurrent attention to PSTs' visions of task implementation.

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